

Scanning x-ray microscopy

Chris Jacobsen

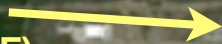
**Argonne Distinguished Fellow, Advanced Photon
Source, Argonne National Laboratory**

**Professor, Physics & Astronomy; Applied
Physics; Chemistry of Life Processes Institute;
Northwestern University**

Chicagoland



**Argonne Leadership
Computing Facility (ALCF)**



**EMC: Electron
Microscopy Center**



**CNM: Center for
Nanoscale Materials**



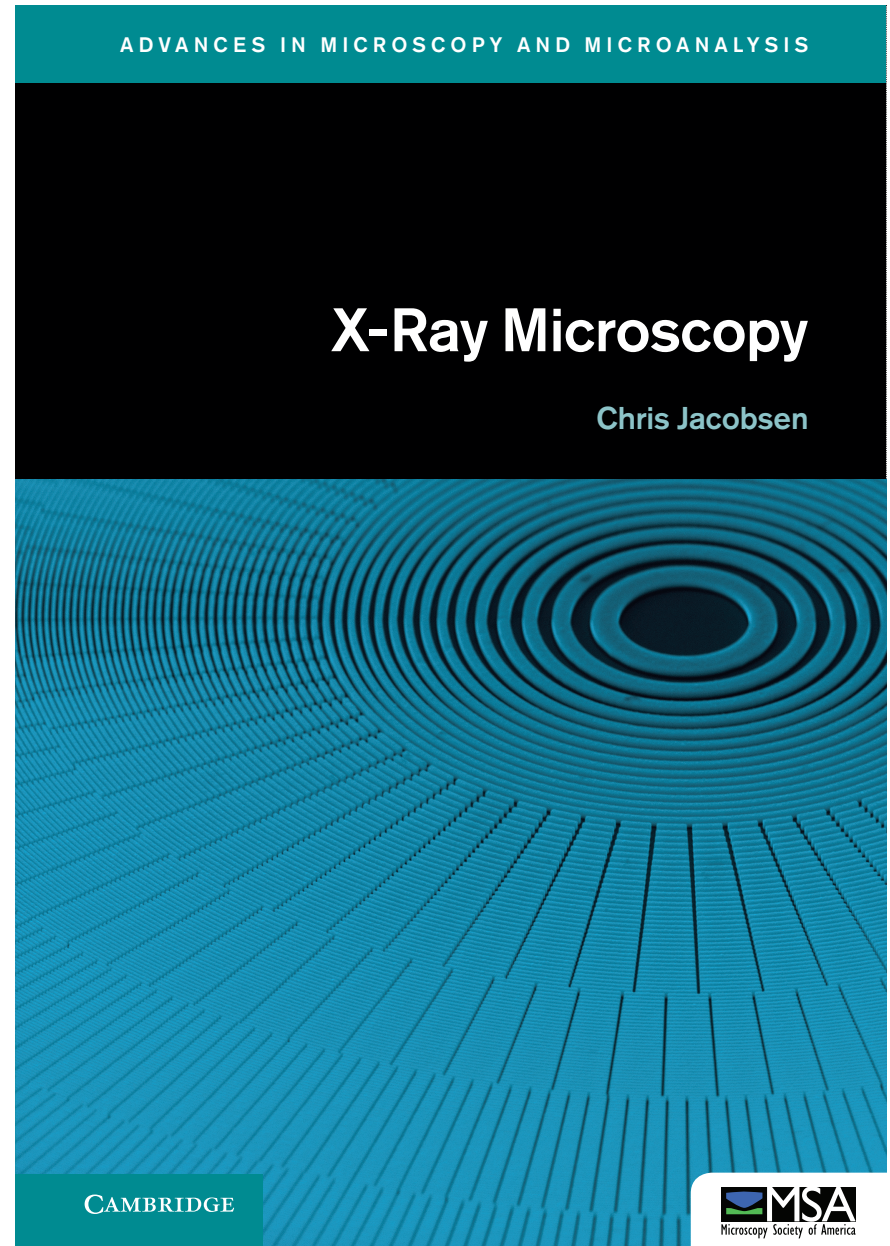
**Advanced
Photon Source**

**APCF: Advanced Protein
Crystallization Facility**



X-ray Microscopy

- www.cambridge.org/Jacobsen
- 580 pages
- December 2019? January 2020?



Outline

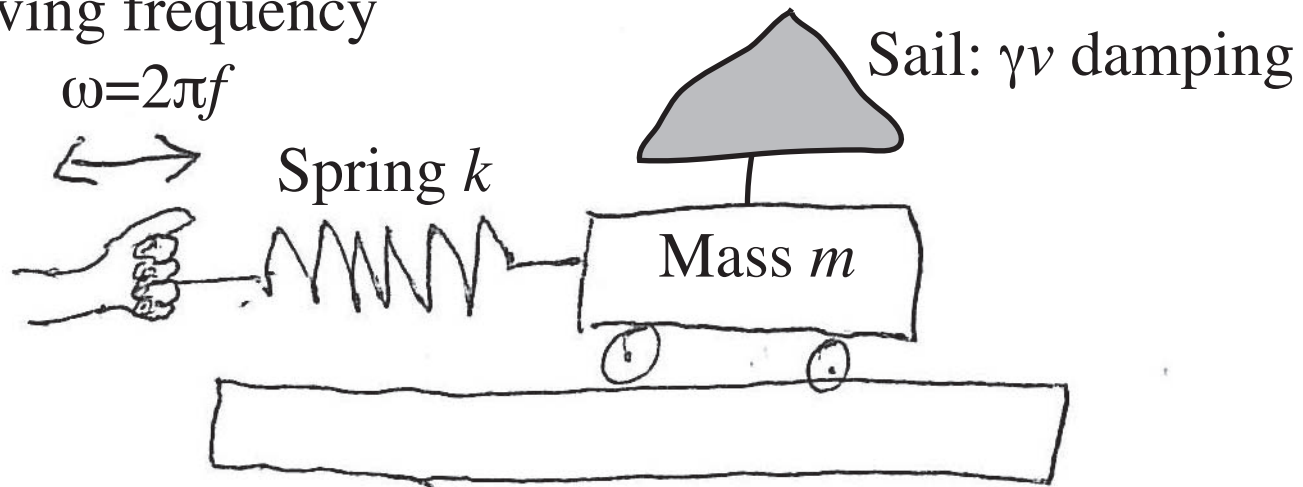
- **X-ray optics and x-ray lenses**
- Why scanning?
- Illumination considerations
- Instrumentation considerations
- Absorption spectromicroscopy
- CDI and ptychography



The refractive index

- Damped, driven harmonic oscillator

Driving frequency



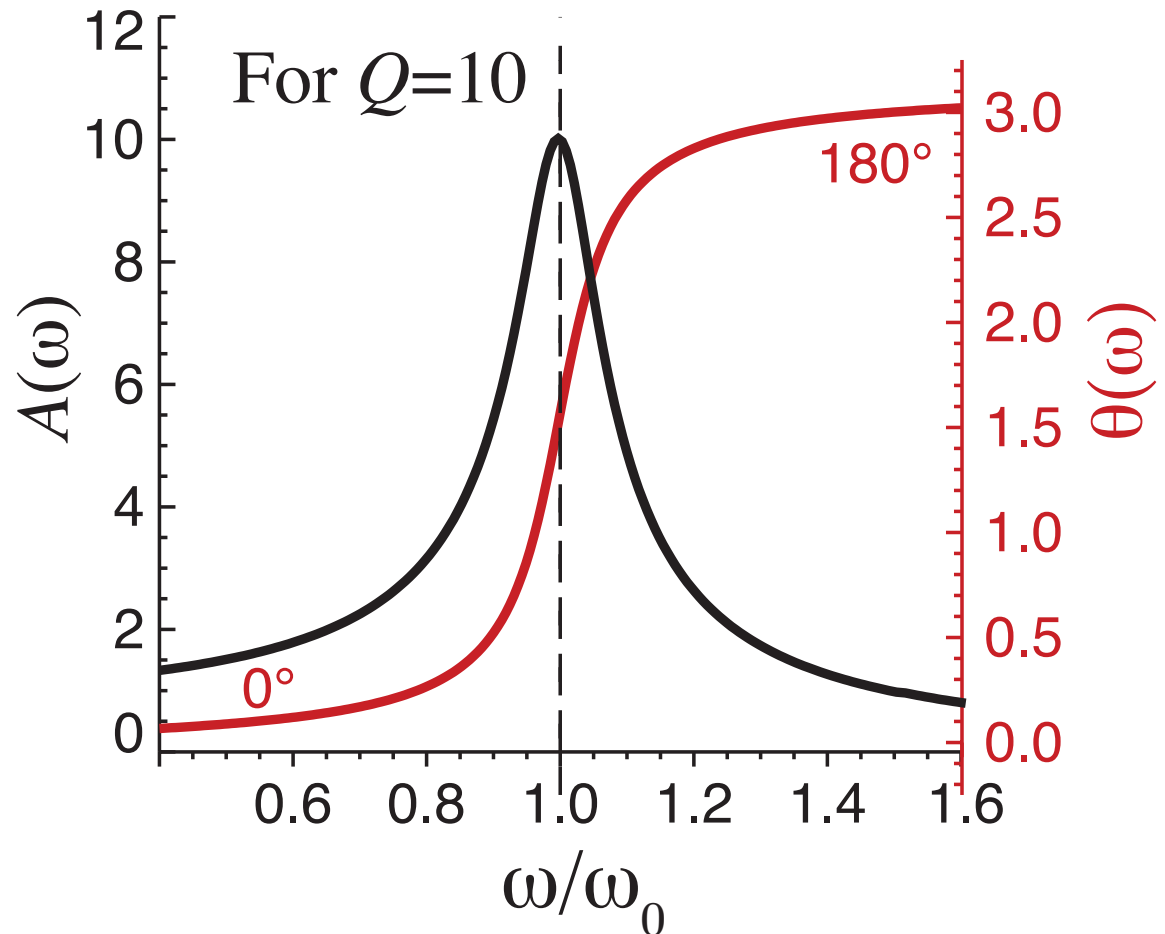
- Damped: scattering, absorption
- Driven: incident electromagnetic wave ω
- Harmonic oscillator: electronic quantum state with energy

$$\hbar\omega = \hbar\sqrt{k/m}$$

Damped, driven harmonic oscillator

- Single resonance: absorption peak, **phase shift across resonance**

$$Q = \omega_0 / \gamma$$

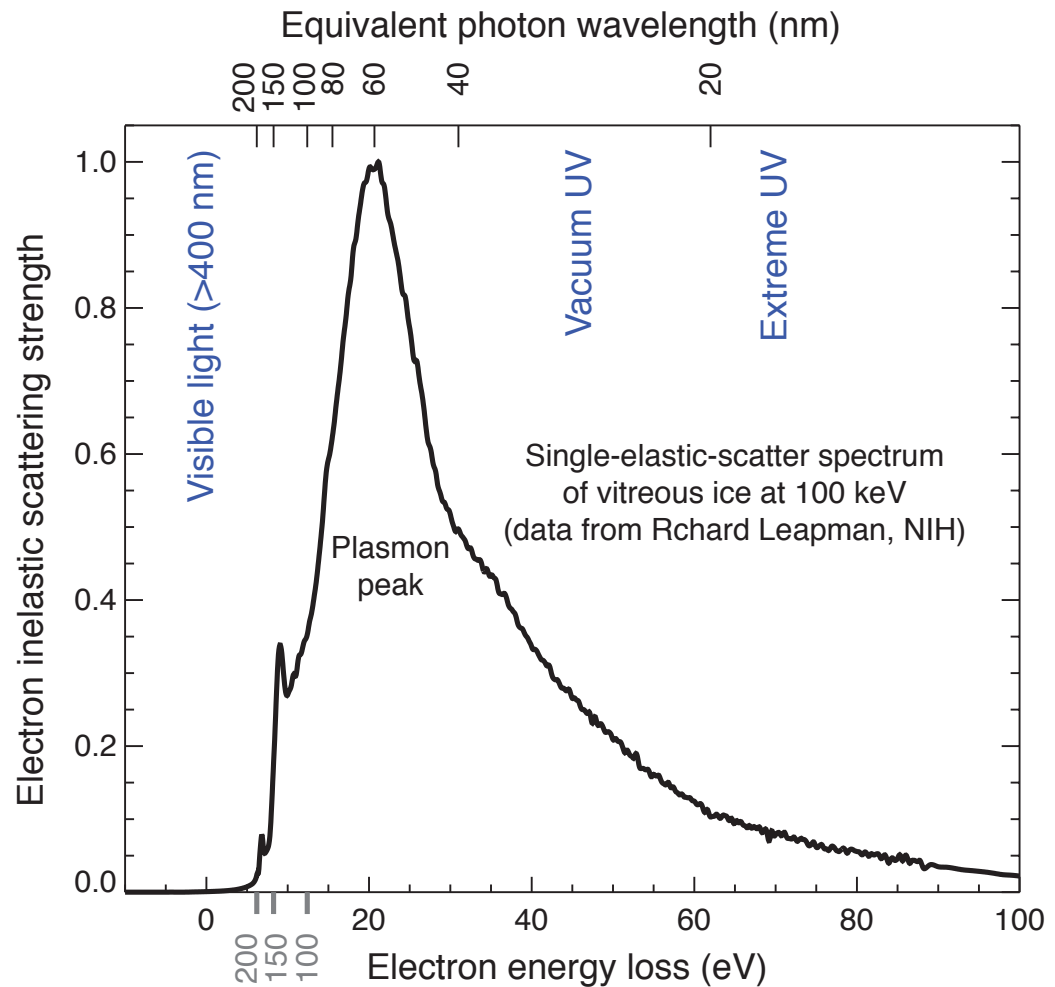


X-rays: the high frequency limit?

What's the dividing line between low and high frequency limits of refractive index? At what frequency are most of the oscillators?

Plasmon frequency

$$\omega_p = (4\pi c^2 n_a r_e)^{1/2}$$



Mysteries of the x-ray refractive index

Write refractive index as

$$n = 1 - \frac{n_a r_e}{2\pi} \lambda^2 (f_1 + i f_2)$$
$$= 1 - \alpha \lambda^2 (f_1 + i f_2)$$

where $n_a = \#$ atoms/volume, and

$r_e = 2.818 \times 10^{-15}$ m is the classical radius of the electron. Assumes $\exp[-i(kx - \omega t)]$ for forward propagation.

Also written as $n = 1 - \delta - i\beta$

Phase velocity is

$$v_p = \frac{\omega}{k} \simeq c(1 + \alpha \lambda^2 f_1)$$

Group velocity is

$$v_g = \frac{d\omega}{dk} \simeq c(1 - \alpha \lambda^2 f_1)$$

Lassen sich Brechungsexponenten der Körper für Röntgenstrahlen experimentell ermitteln?

Von A. Einstein.

(Eingegangen am 21. März 1918.)

Vor einigen Tagen erhielt ich von Herrn Prof. A. KÖHLER (Wiesbaden) eine kurze Arbeit¹⁾, in welcher eine auffallende Erscheinung bei Röntgenaufnahmen geschildert ist, die sich bisher nicht hat deuten lassen. Die reproduzierten Aufnahmen — zu meist menschliche Gliedmaßen darstellend — zeigen an der Kontur einen hellen Saum von etwa 1 mm Breite, in welchem die Platte heller bestrahlt zu sein scheint als in der (nicht beschatteten) Umgebung des Röntgenbildes.

Ich möchte die Fachgenossen auf diese Erscheinung hinweisen und beifügen, daß die Erscheinung wahrscheinlich auf Totalreflexion beruht. Nach der klassischen Dispersionstheorie müssen wir erwarten, daß der Brechungsexponent n für Röntgenstrahlen nahe an 1 liegt, aber im allgemeinen doch von 1 verschieden ist. n wird kleiner bzw. größer als 1 sein, je nachdem der Einfluß derjenigen Elektronen auf die Dispersion überwiegt, deren Eigenfrequenz kleiner oder größer ist als die Frequenz der Röntgenstrahlen. Die Schwierigkeit einer Bestimmung von n liegt darin, daß $(n - 1)$ sehr klein ist (etwa 10^{-5}). Es ist aber leicht einzusehen, daß bei nahezu streifender Inzidenz der Röntgenstrahlen im Falle $n < 1$ eine nachweisbare Totalreflexion auftreten muß.

X-ray refractive index

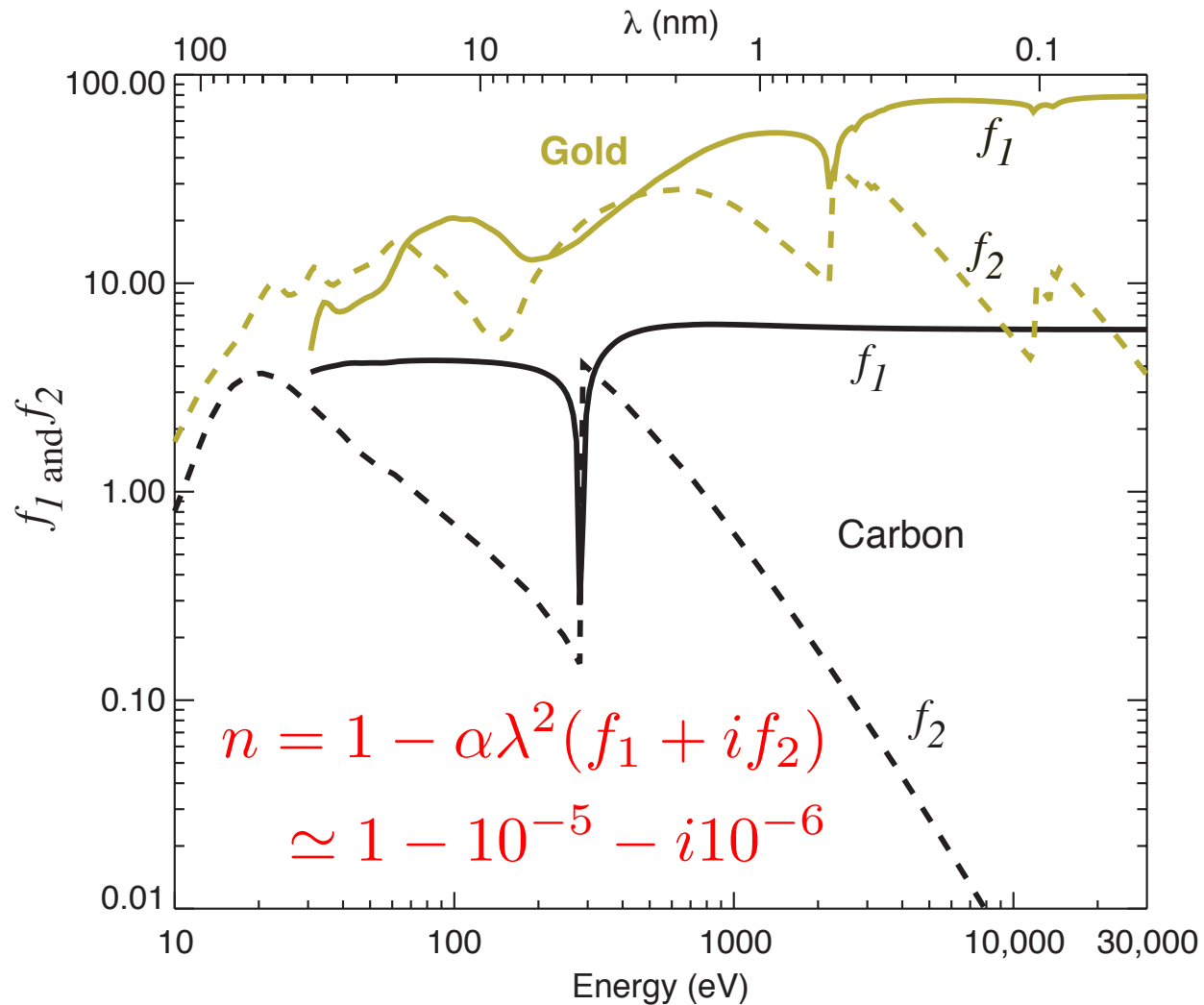
Refractive index of
 $n = 1 - \alpha \lambda^2 (f_1 + i f_2)$

Real part of oscillator strength f_1 tends towards atomic number Z

Imaginary part of oscillator strength f_2 declines as E^{-2}

Phase $\exp[-ink]$ is advanced relative to vacuum by $2\pi\alpha\lambda f_1$

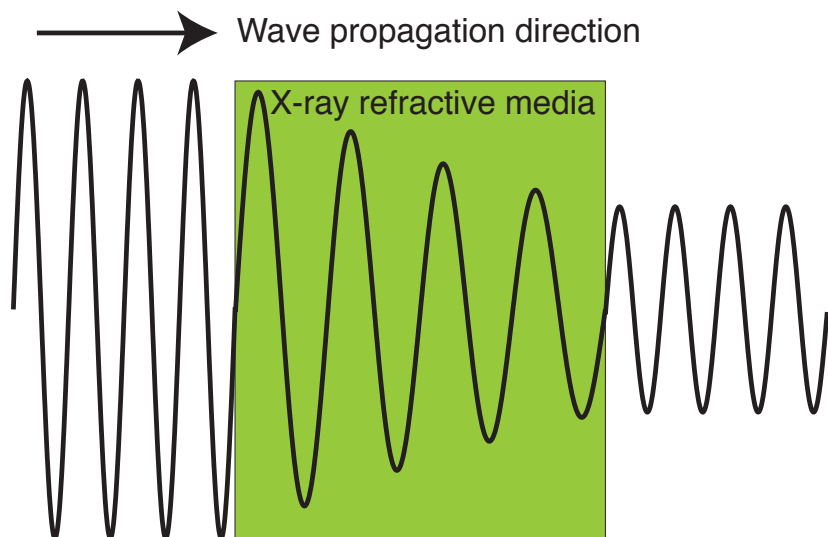
Intensity is decreased as $\exp[-4\pi\alpha\lambda f_2]$



Data from http://henke.lbl.gov/optical_constants/

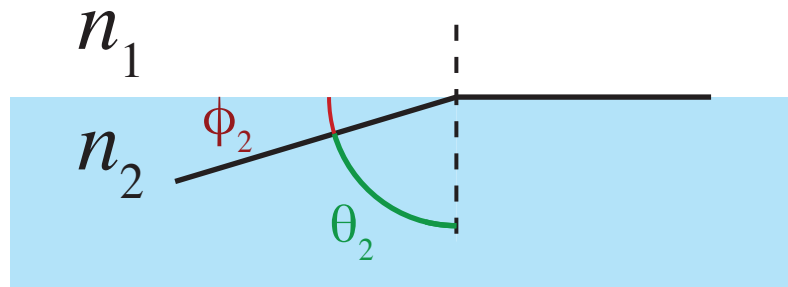
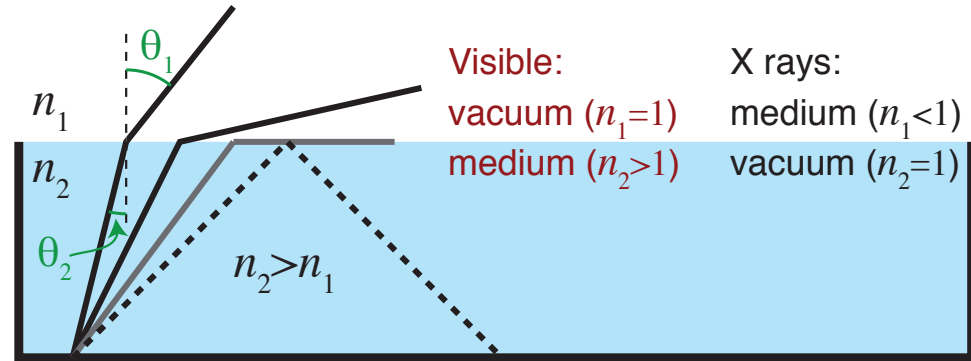
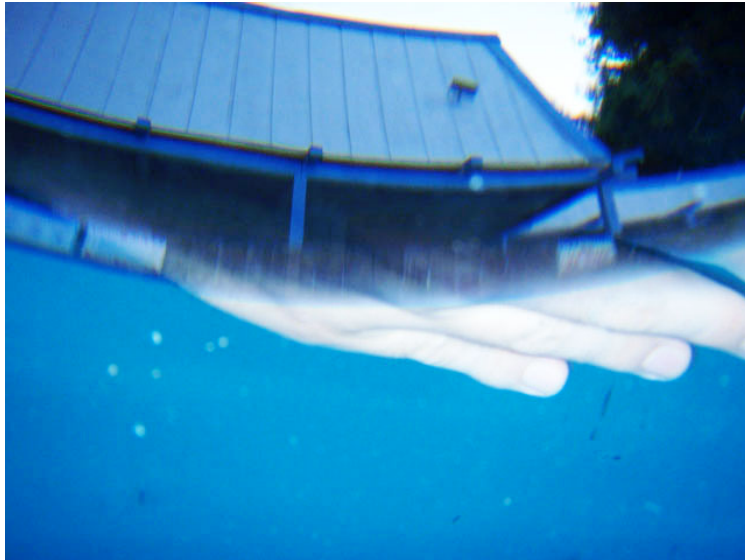


X rays in media



X-ray mirrors use total *internal* reflection!

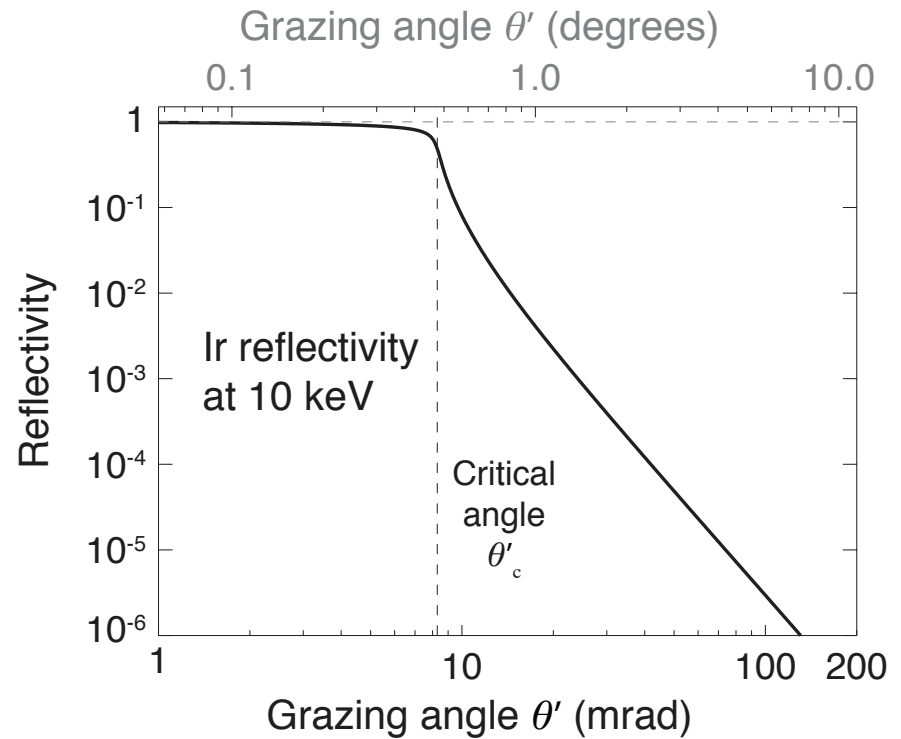
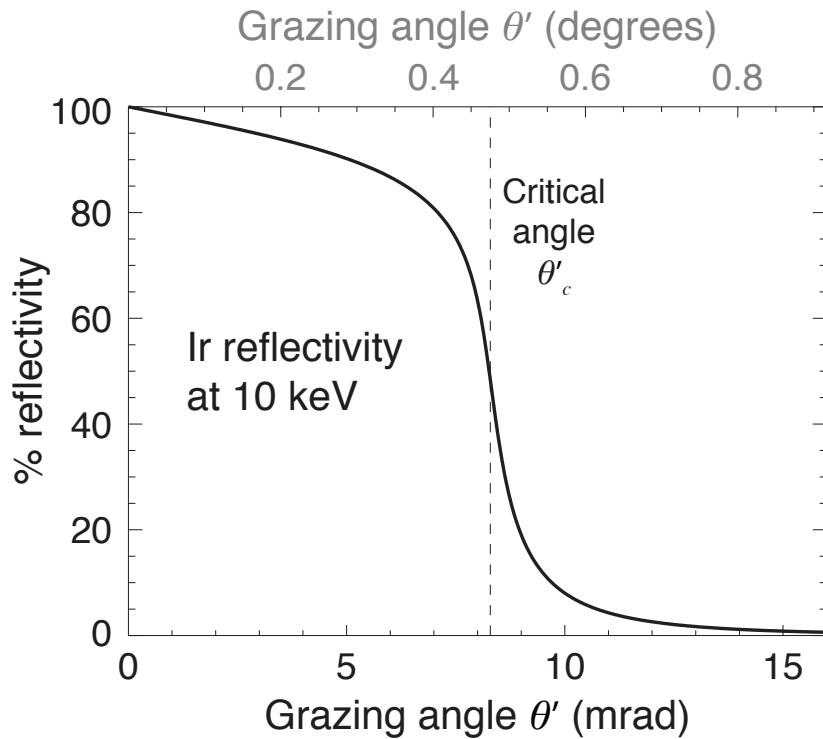
- Total internal reflection happens when $\theta_1=90^\circ$ in $n_1\sin\theta_1=n_2\sin\theta_2$, or $\theta_2=\text{asin}(n_1/n_2)$.



- Switch from angle θ_2 relative to normal incidence, to angle ϕ_2 relative to grazing incidence, or $\sin(\theta_2)=\cos(90^\circ-\phi_2)=\sin(\phi_2)$
- We then have $n_1=n_2\sin(\phi_2)$ or with $n_2=1$ and $\sin(\phi_2)\approx 1-(\phi_2)^2/2$ we have a **grazing incidence critical angle of $\phi_2 = \lambda(2\alpha f_1)^{1/2}$**
- Note diffraction resolution limit of ϕ_2/λ is (almost) independent of wavelength!



X-ray mirror reflectivity



Lord Rayleigh (John William Strutt) and telescopes

- Diffraction from a circular aperture limits the angular resolution of telescopes
- Diffraction is described by the Airy function with a first minimum at $\nu=1.22\pi$:

$$\left[2J_1(\nu)/\nu\right]^2$$

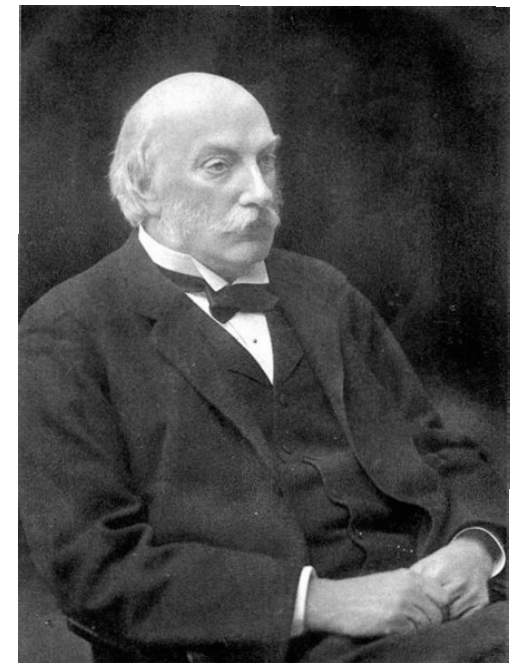
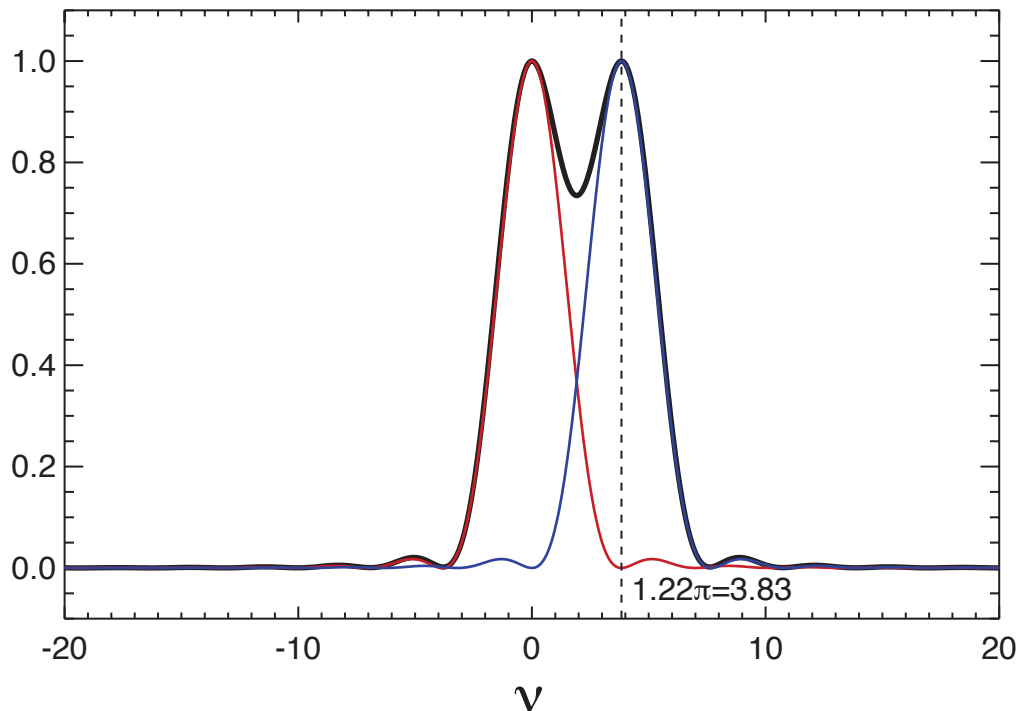
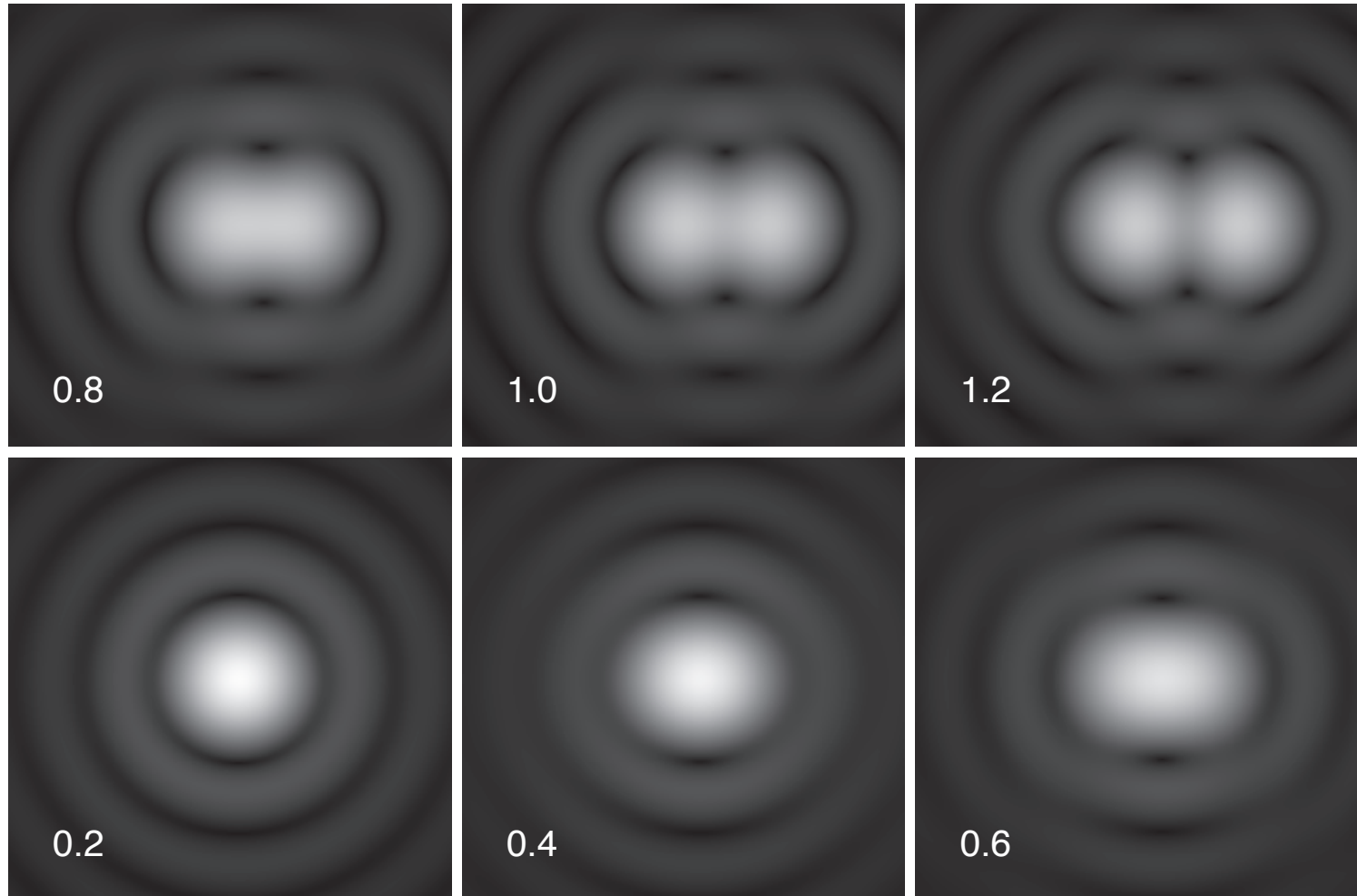


Illustration of Rayleigh resolution

- Separation of two spots as a fraction of the Rayleigh resolution



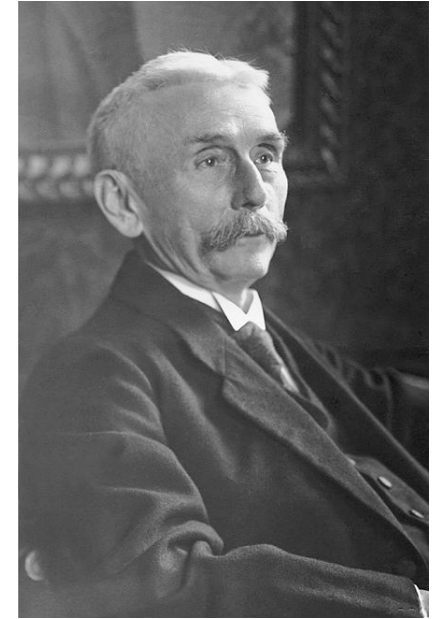
Ernst Abbe and Carl Zeiss, plus Otto Schott



Ernst Abbe



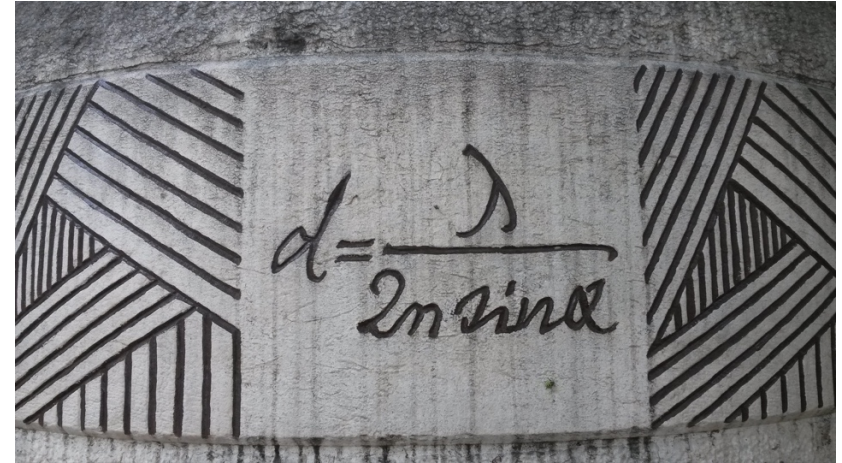
Carl Zeiss



Otto Schott

ZEISS

Microscope resolution: Rayleigh, plus index n



d =resolution

n =refractive index of medium
between lens and specimen

θ =semi-angle of lens

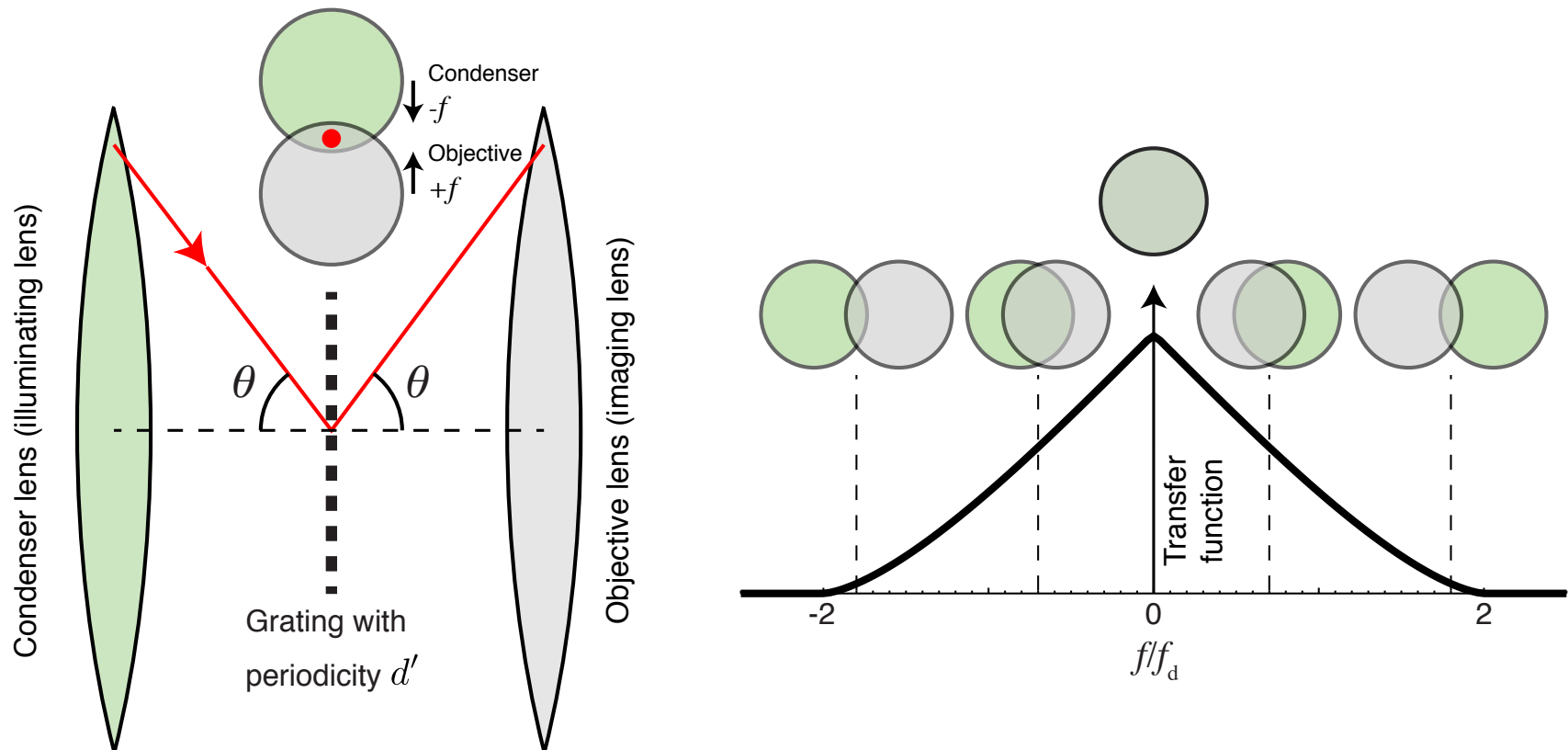
Microscope resolution: Rayleigh, plus index n



Abbey Road by The Beatles

Modulation transfer function for incoherent brightfield

- Lens semi-angle θ ($NA=n\sin\theta$): $\theta=\lambda/d'$
- Coherent imaging cutoff at spatial frequency of $f_d=1/d'$
- Incoherent imaging goes to twice the spatial frequency, but with losses



Paul Kirkpatrick and Albert Baez, 1948

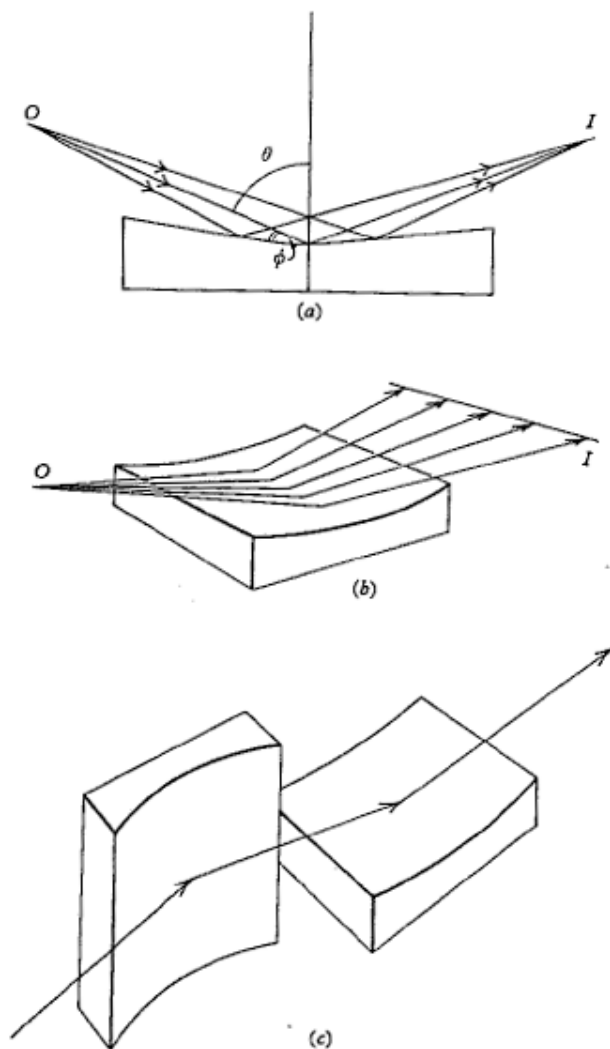


Fig. 1.2. Reflexion X-ray microscopy. (a) and (b) X-rays diverging from a source O are focused by a cylindrical surface to form an astigmatic image I ; (c) arrangement of two cylindrical mirrors for eliminating astigmatism. (Kirkpatrick & Pattee, 1953.)

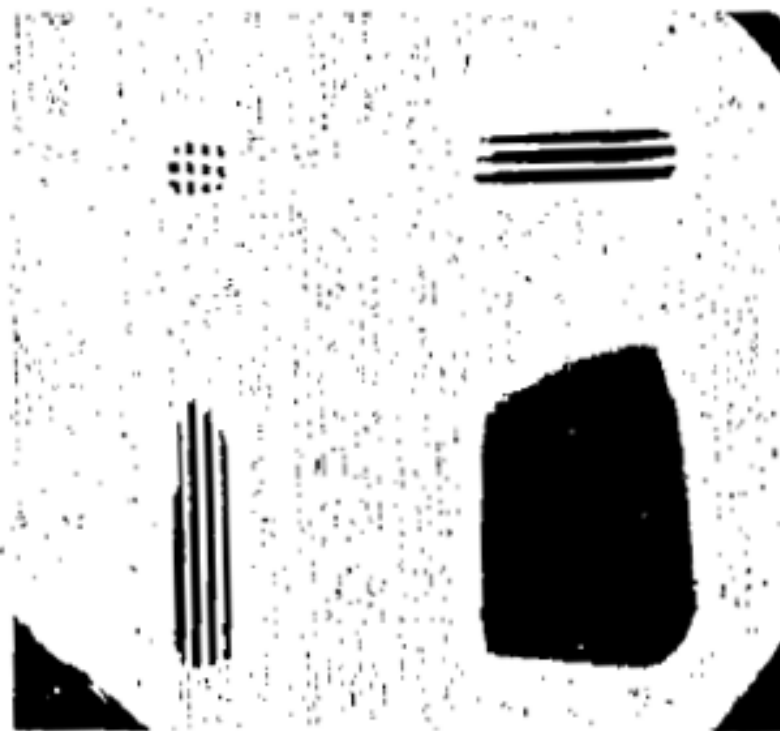


FIG. 12. Pattern produced by mirrors arranged as in Fig. 11. Object was a monel screen having 350 meshes per linear inch. In addition to the full image of the screen two partial images, each formed by one mirror, and a large spot caused by direct radiation appear above.

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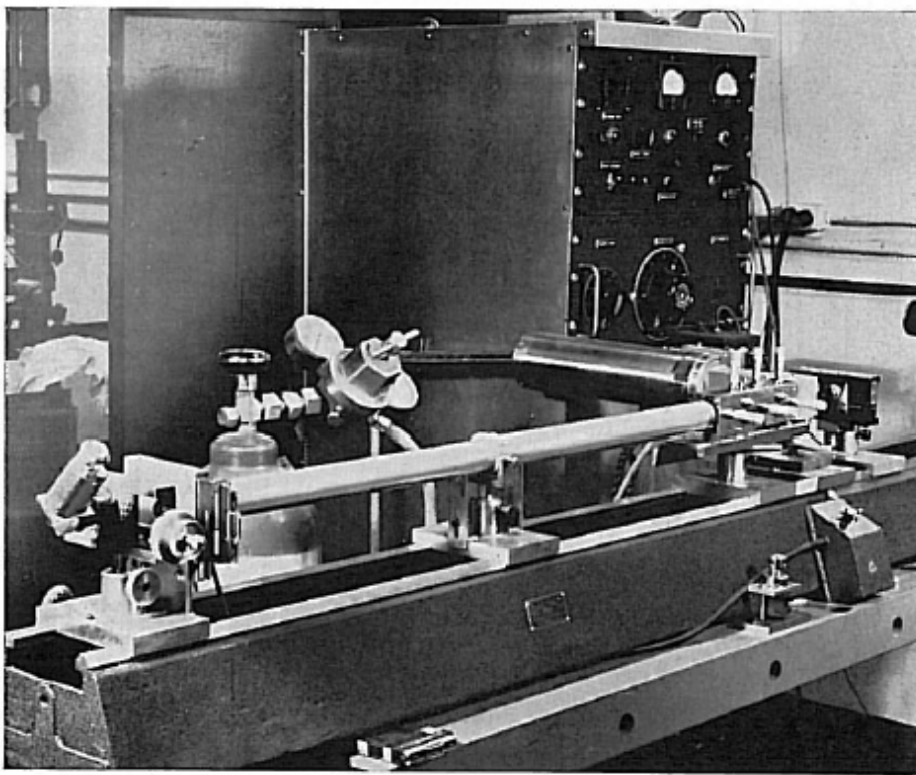
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THE X-RAY MICROSCOPE

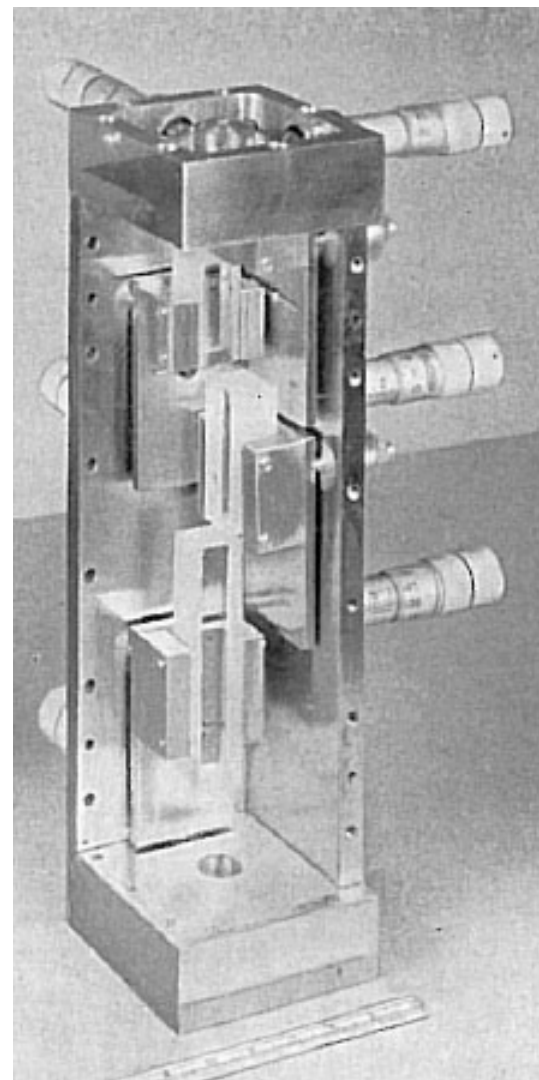
by Paul Kirkpatrick

It would be a big improvement on microscopes using light or electrons, for X-rays combine short wavelengths, giving fine resolution, and penetration. The main problems standing in the way have now been solved. 44

Kirkpatrick and Pattee, 1953

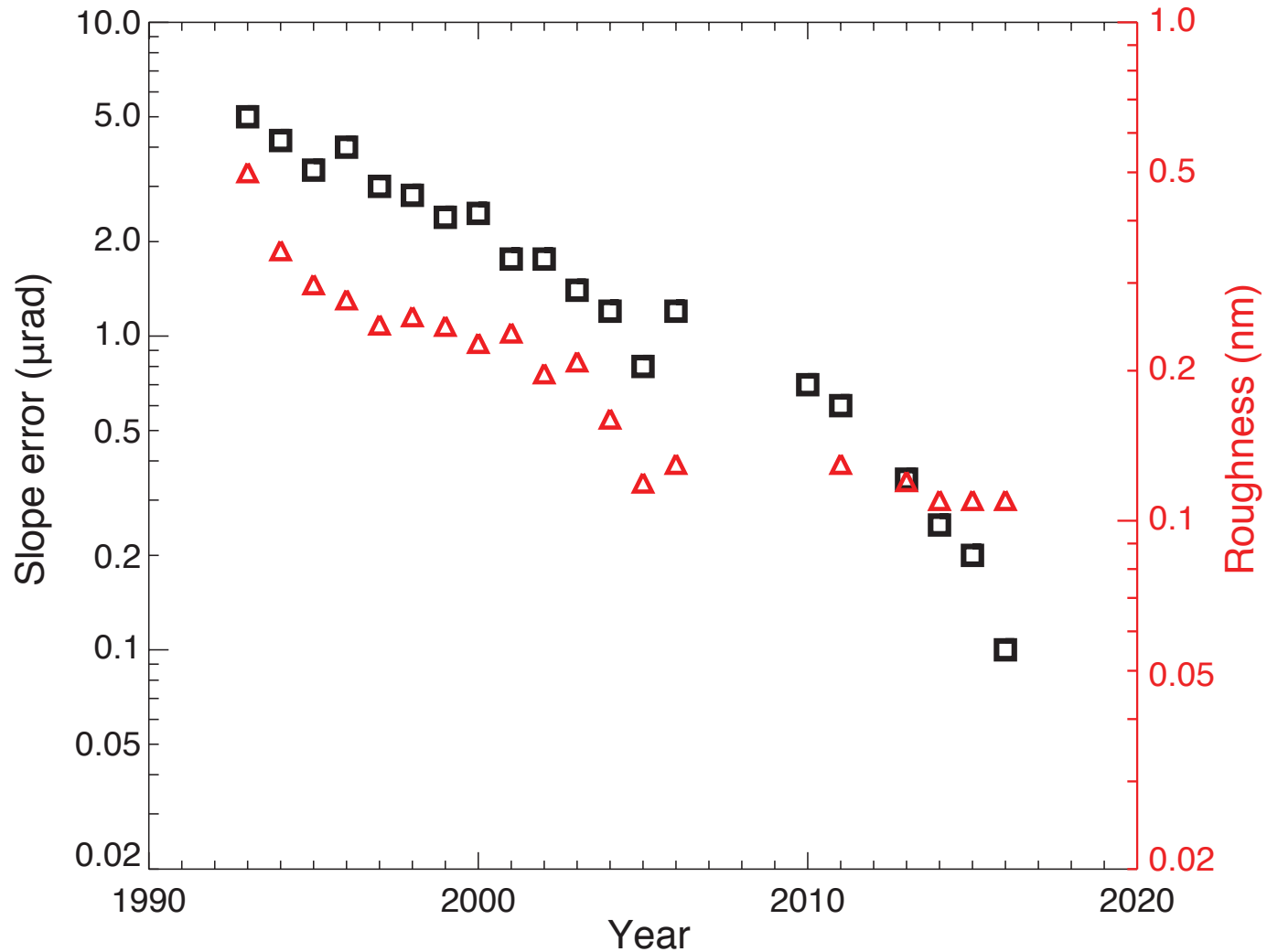


Reflexion X-ray microscope of Kirkpatrick and Pattee, incorporating two pairs of mirrors. (Kirkpatrick & Pattee, 1953.)



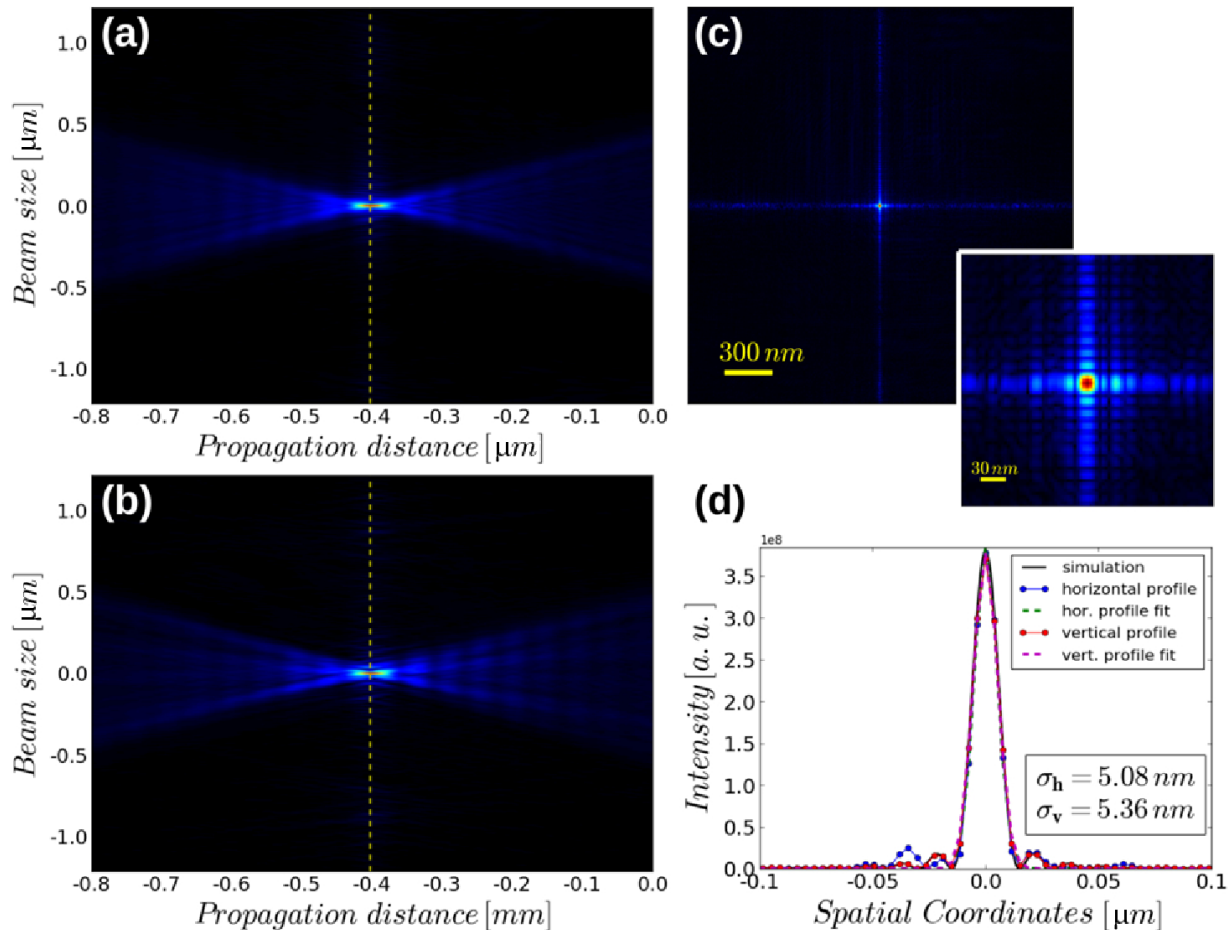
Improving the figure and roughness of mirrors

- Data provided by Lahsen Assoufid, head of the Optics Group at the APS

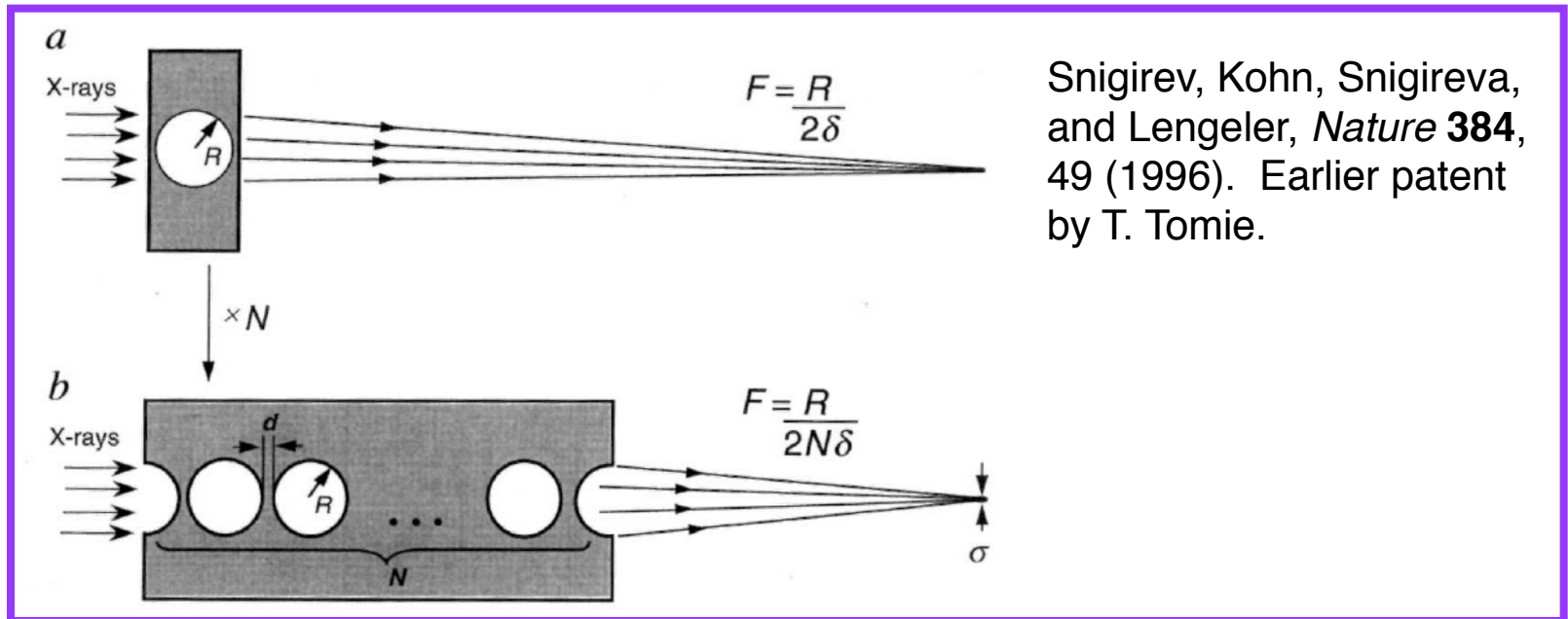
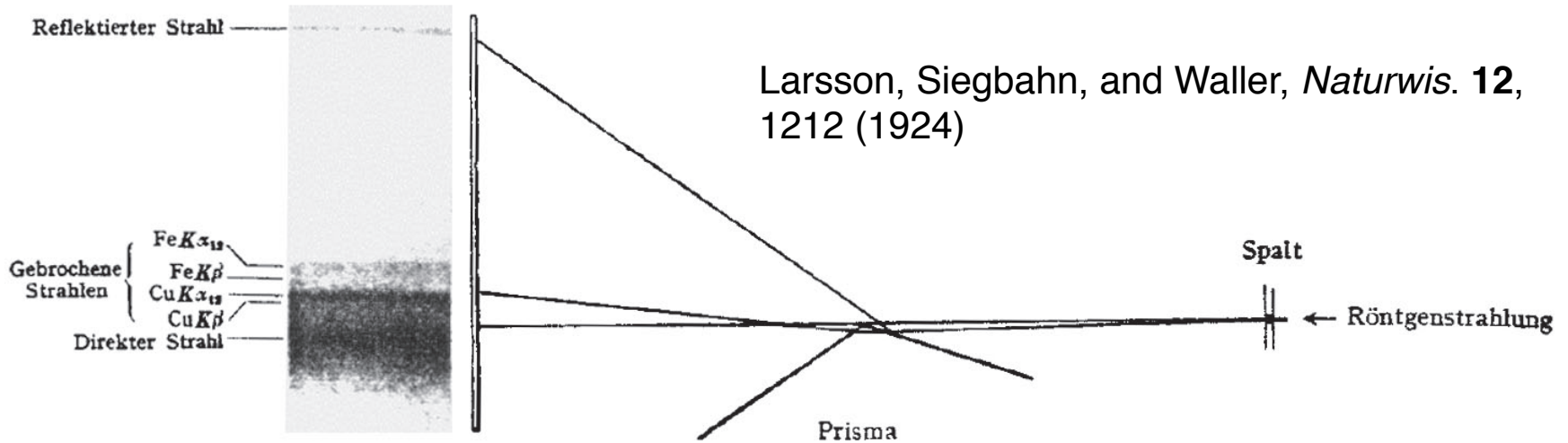


Nanofocusing with multilayer-coated KB mirrors

- da Silva, Pacureanu, Yang, Bohic, Morawe, Barrett, and Cloetens, *Optica* 4, 492 (2017)
- FWHM of 12.0 nm (H) and 12.6 nm (V) at 17 keV

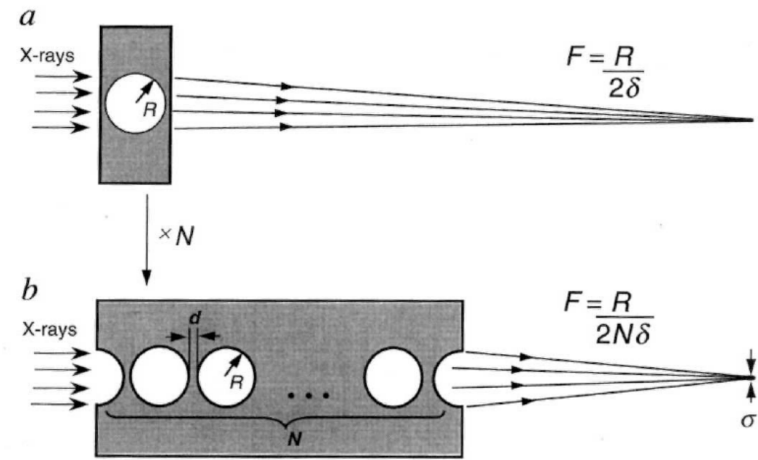


X-ray refraction

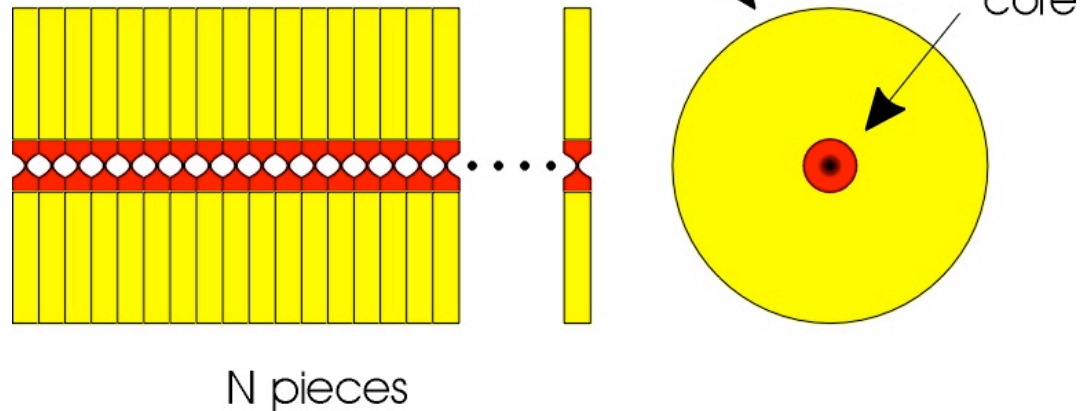


Compound refractive lenses

- Röntgen tried to make lenses, but found no focusing.
- Focal length of one lens is long – so combine many lenses! Tomie; Snigirev *et al.*, *Nature* **384**, 49 (1996); Lengeler *et al.*, *J. Synch. Rad.* **9**, 119 (2002).
- Resolution approaching 60 nm at 5-10 keV with parabolic beryllium lenses.
- Refractive lenses are especially good at 20 keV and higher.

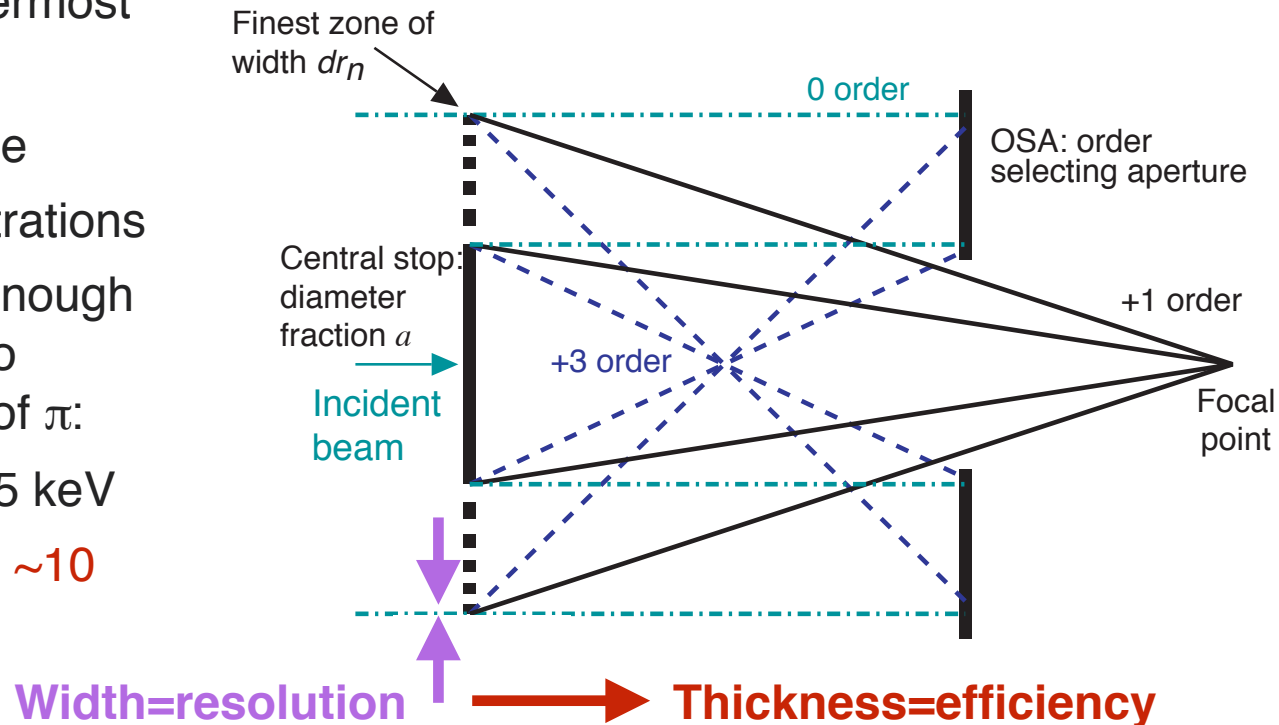


Compound refractive lenses at Universität Aachen



X-ray focusing: Fresnel zone plates

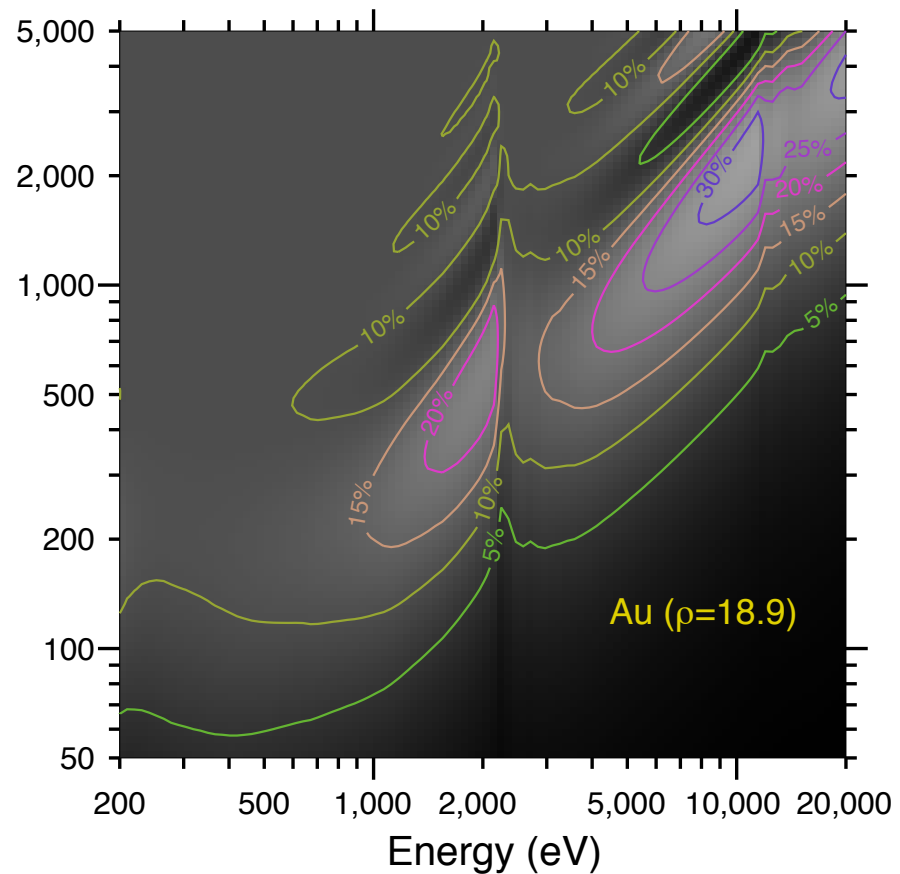
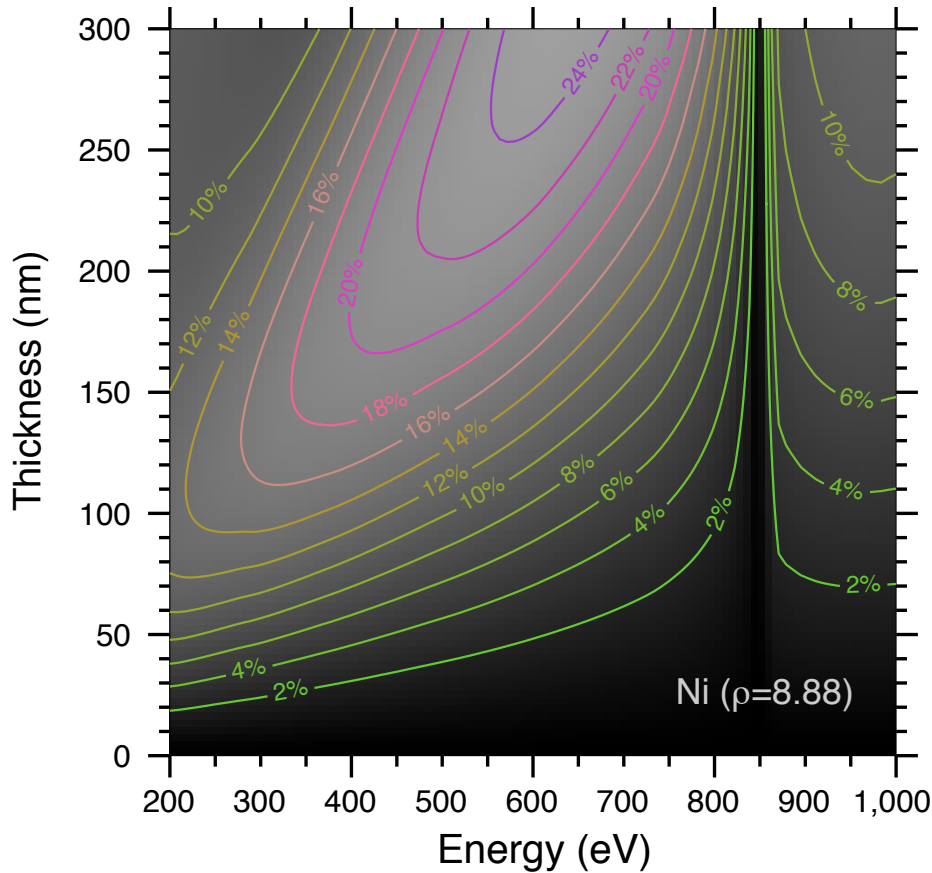
- Diffractive optics: radially varied grating spacing
- Spatial resolution limited to **width** dr_N of finest, outermost zone.
 - 20-40 nm in practice
 - <10 nm in demonstrations
- Zones must be **thick** enough along beam direction to produce a phase shift of π :
 - about 100 nm at 0.5 keV
 - **several 1000 nm at ~10 keV!**



High aspect ratio nanofabrication!

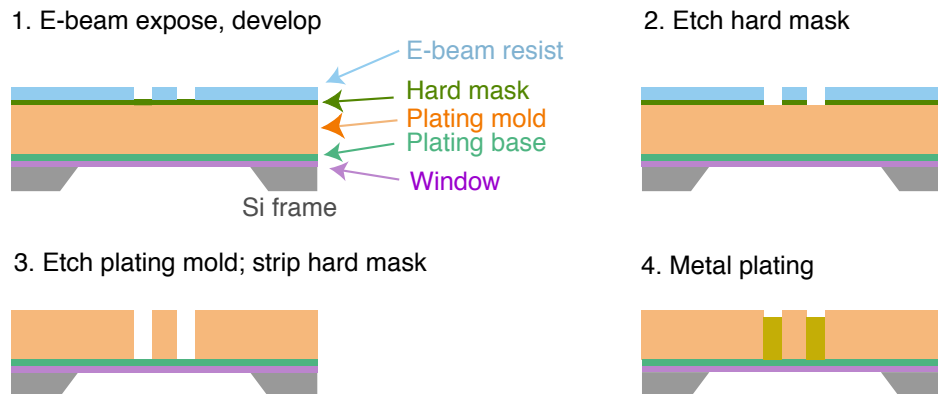
Zone plate efficiency and thickness

For binary zones, 1:1 mark:space ratio.
See Kirz, *J. Opt. Soc. Am.* **64**, 301 (1974)



Zone plates by electron beam lithography

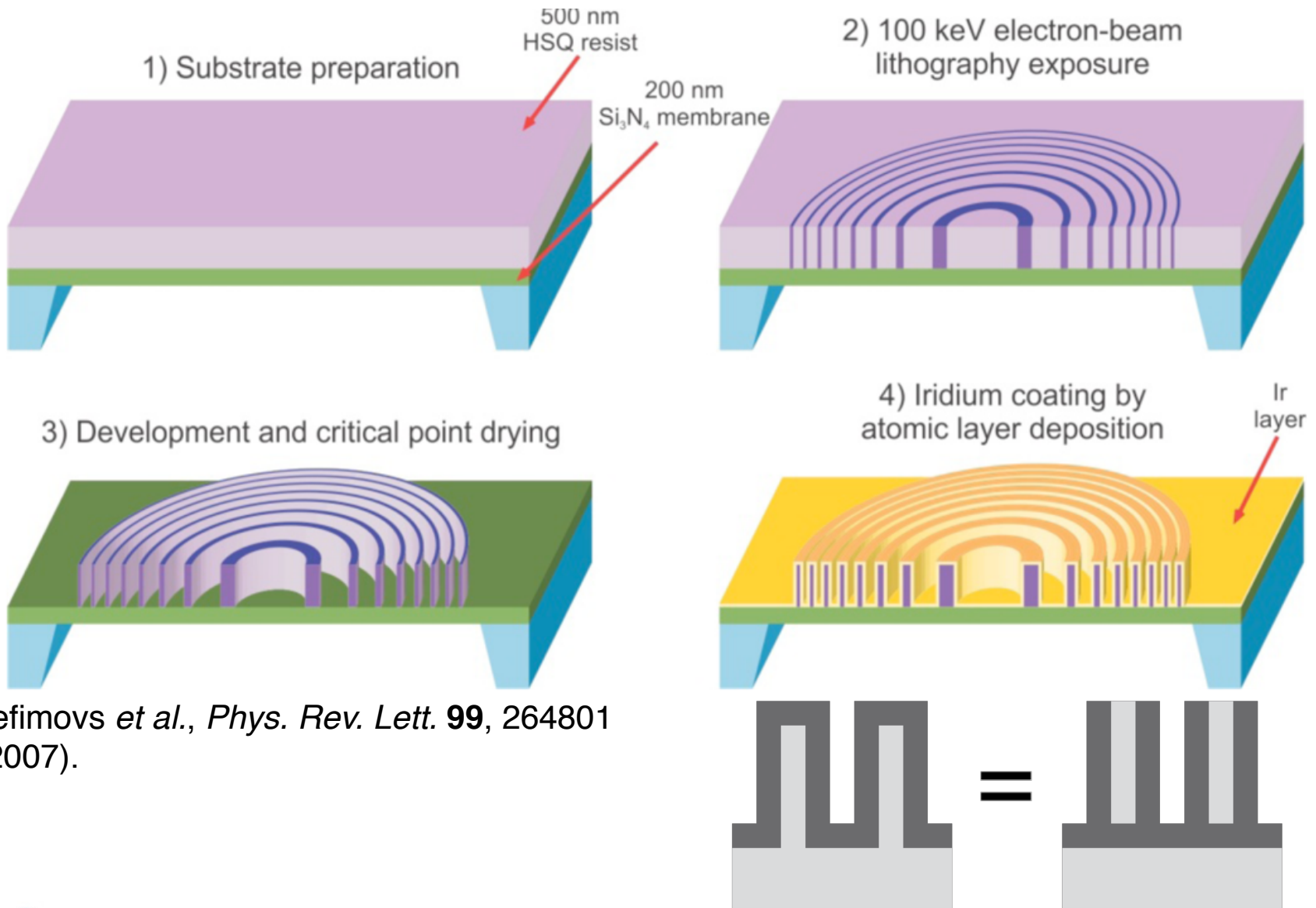
- Electron beam lithography: produces the finest possible structures (other than what nature can be persuaded to make by itself)
 - Example: JEOL JBX-9300FS: 1 nA into 4 nm spot, 1.2 nm over 500 μm , 100 keV
- Electrons scatter within resist, so highest resolution is only within ~ 100 nm thickness.
- Use directional etching methods like reactive ion etching for thick structures



A. Stein and JBX-9300FS



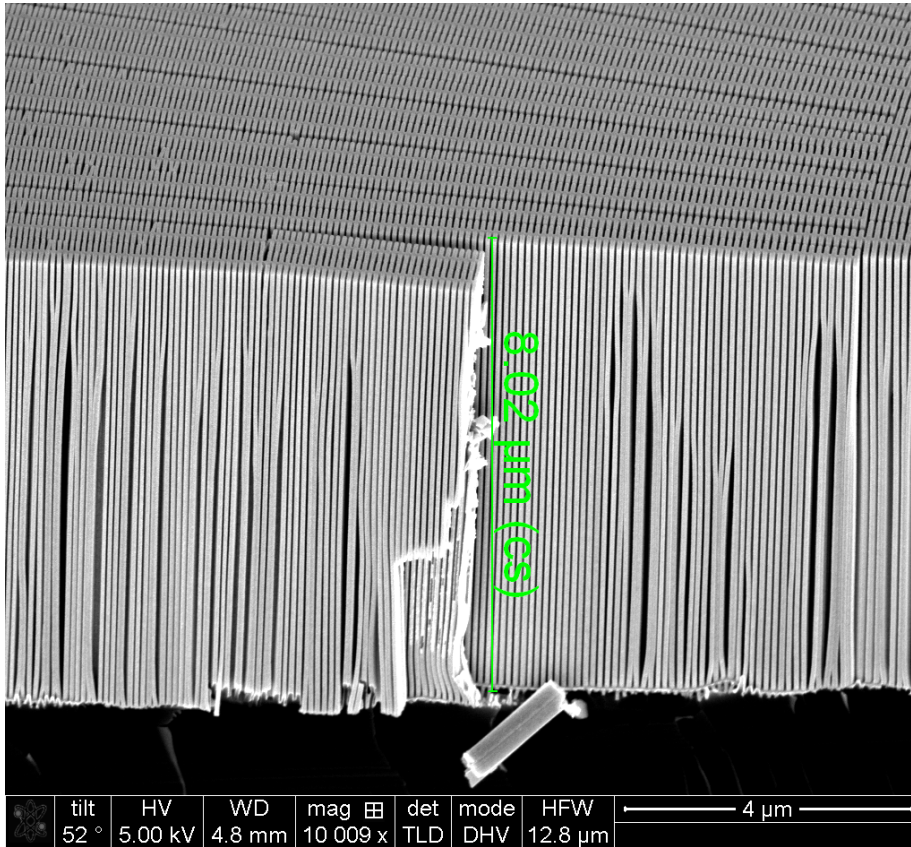
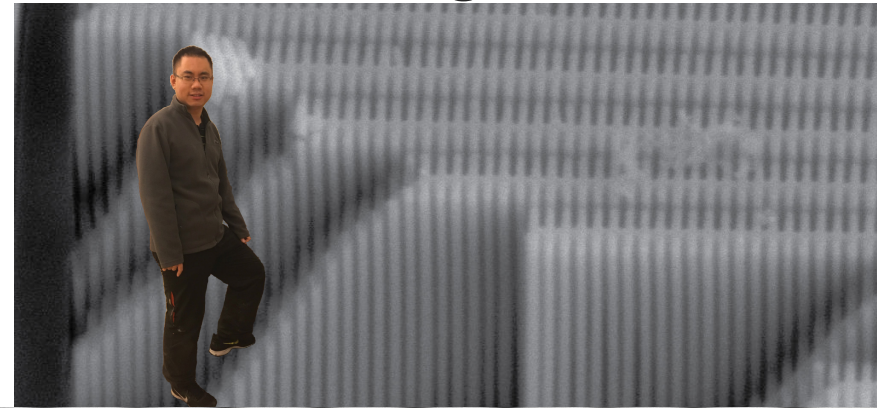
Improving single zone plates: zone doubling



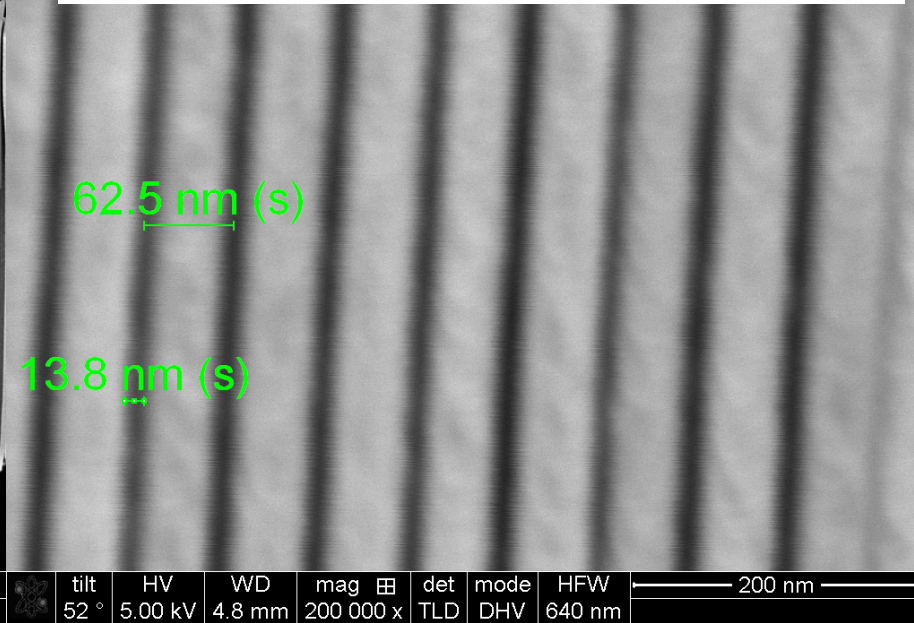
Fresnel zone plates for x-ray nanofocusing

14 nm zone width in Pt, up to 8 μm tall (aspect ratio=500). 6% efficient at 20 keV in preliminary tests; resolution tests underway.

Kenan Li, M. Wojcik, R. Divan, L. Ocola, B. Shi, D. Rosenmann, and C. Jacobsen, *J. Vac. Sci. Tech. B* (Nov. 2017)



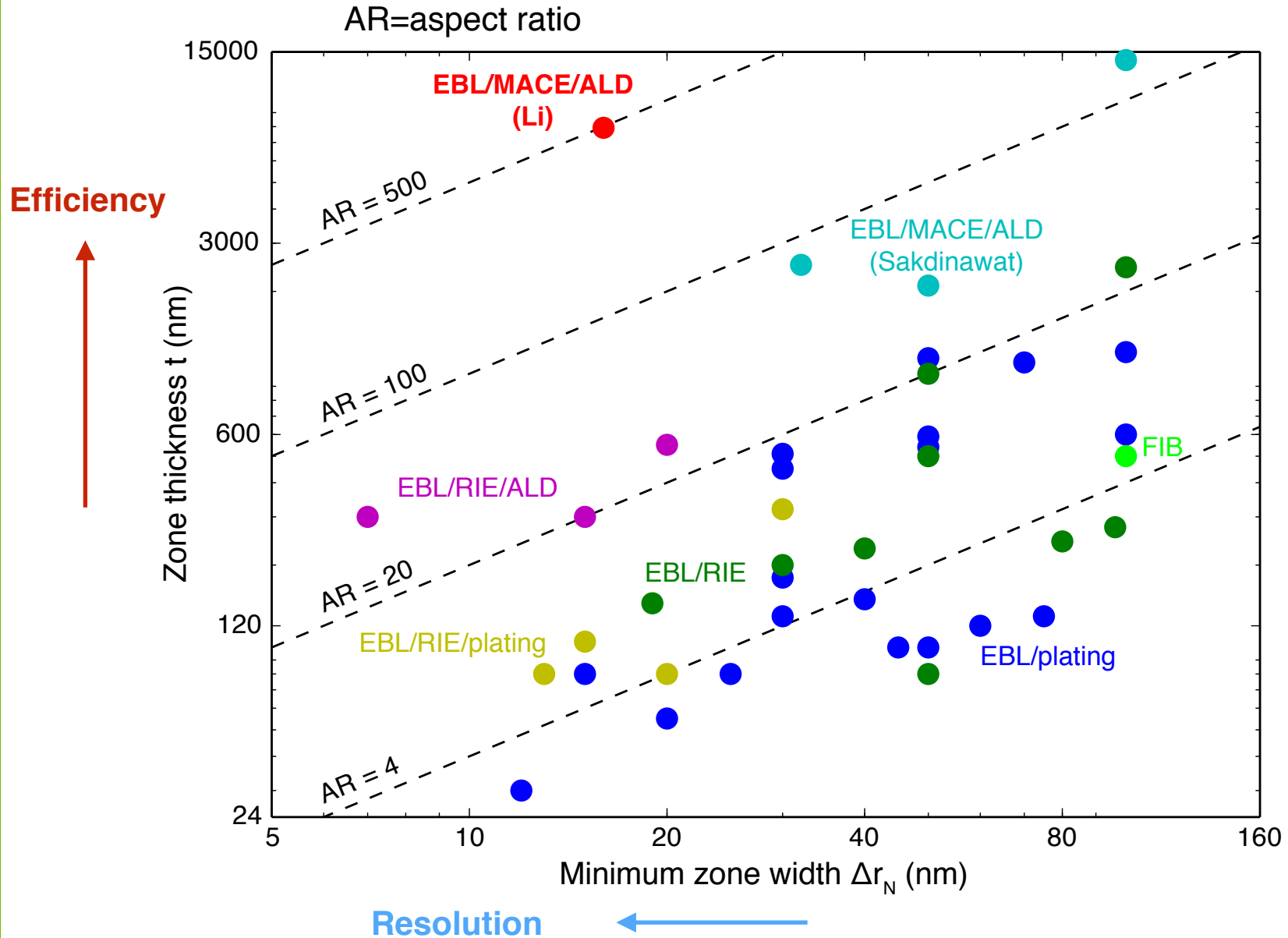
Metal-assisted chemical etching of silicon and atomic layer deposition to produce Pt zones. 14 nm wide zones that are 8 μm tall! Aspect ratio >500



tilt	HV	WD	mag	det	mode	FWHM	4 μm
52 °	5.00 kV	4.8 mm	10 009 x	TLD	DHV	12.8 μm	

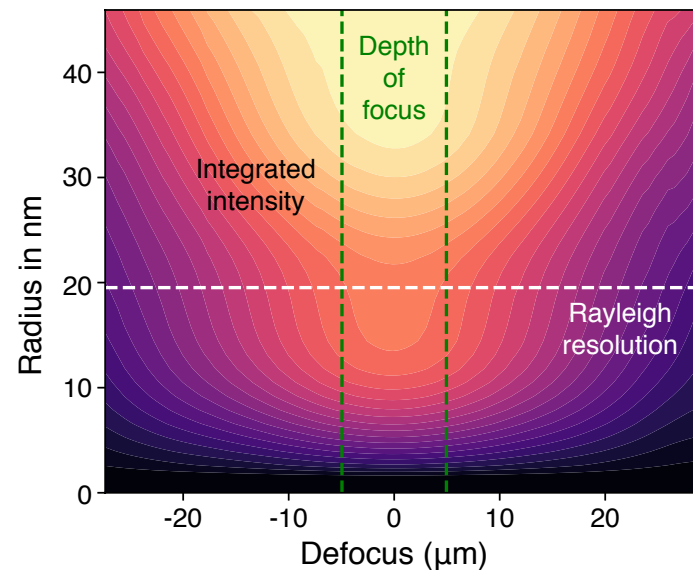
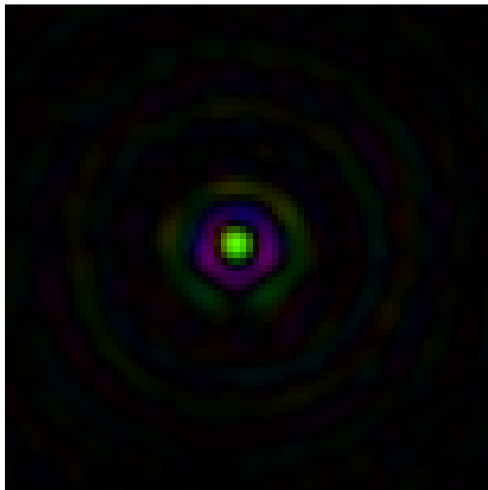
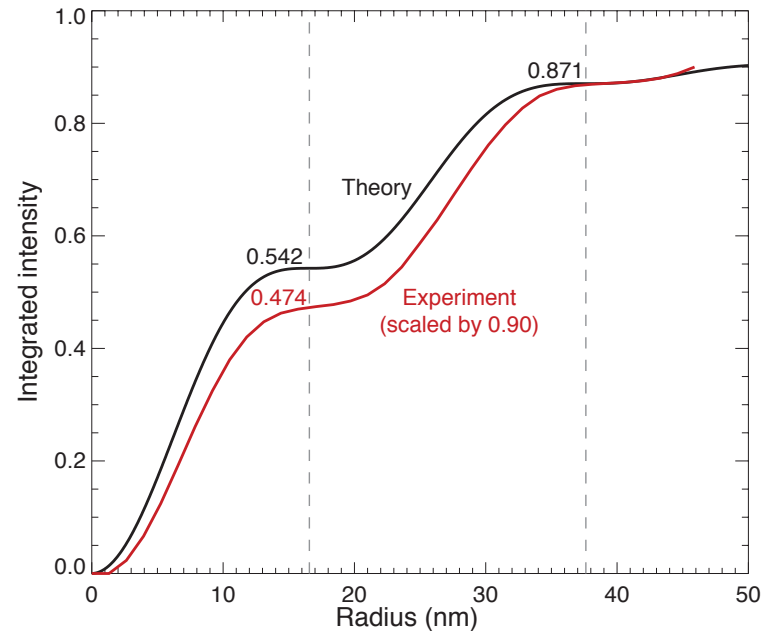
tilt	HV	WD	mag	det	mode	FWHM	200 nm
52 °	5.00 kV	4.8 mm	200 000 x	TLD	DHV	640 nm	

Recent APS/CNM/NU results: off the charts, on a log scale!



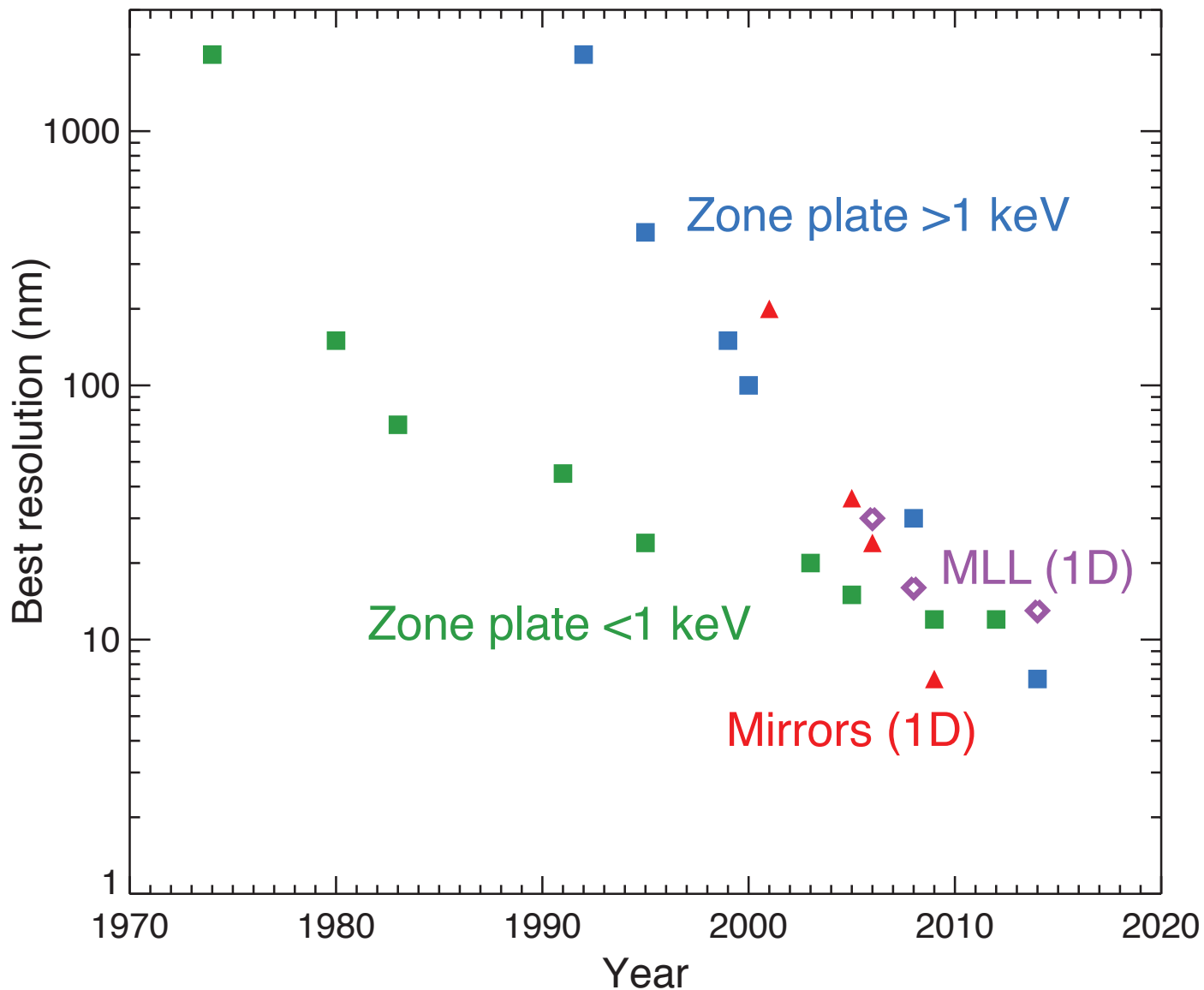
Recent tests at Brookhaven Lab

- 14 nm FWHM probe size at 12 keV, with 10^8 photons/second in the focus
- Northwestern University: Kenan Li, Sajid Ali, Chris Jacobsen
- Argonne Lab: Michael Wojcik
- Brookhaven Lab: Xiaojing Huang, Hanfei Yan, Yong Chu, Ajith Pattammattel, Evgueni Nazaretski



X-ray nanofocusing approaches are improving

X-ray nanofocusing has improved over time - but resolution in practical experiments is still limited to 20-30 nm



Outline

- X-ray optics and x-ray lenses
- **Why scanning?**
- Illumination considerations
- Instrumentation considerations
- Absorption spectromicroscopy
- CDI and ptychography

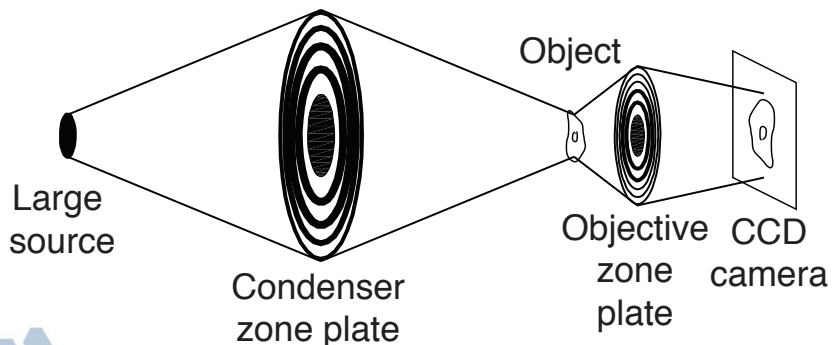


Zone plate microscopes

Full-field: transmission x-ray microscope (TXM)

- Incoherent illumination; works well with a bending magnet or a laboratory source
- Inefficient zone plate is *after* the sample (higher radiation dose)
- Faster (pixels in parallel)
- If zone plate condensers are used as monochromators, poor spectral resolution
- Transmission or reflection imaging

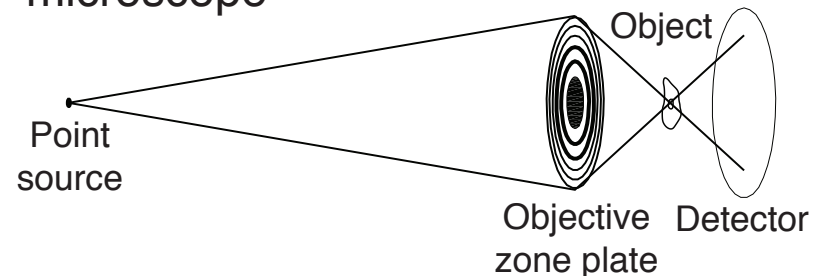
TXM: transmission x-ray microscope



Scanning transmission x-ray microscope (STXM)

- Coherent illumination; works best with an undulator
- Inefficient zone plate is *before* the sample (lower radiation dose)
- Slower (pixels one-by-one)
- Unlimited field of view and magnification (scanning stages)
- Better suited to high resolution monochromators
- Flexible modalities: fluorescence etc.

STXM: scanning transmission x-ray microscope



Outline

- X-ray optics and x-ray lenses
- Why scanning?
- **Illumination considerations**
- Instrumentation considerations
- Absorption spectromicroscopy
- Fluorescence microscopy



Nanofocusing requires coherent illumination

- Full-width, full-angle phase space of a diffraction limited lens with numerical aperture θ : $(2\theta) \cdot (2 \cdot 0.61\lambda / \theta) = 2.44\lambda$
- Thus need to limit source phase space to $\sim\lambda$ both in x and y

Illumination source

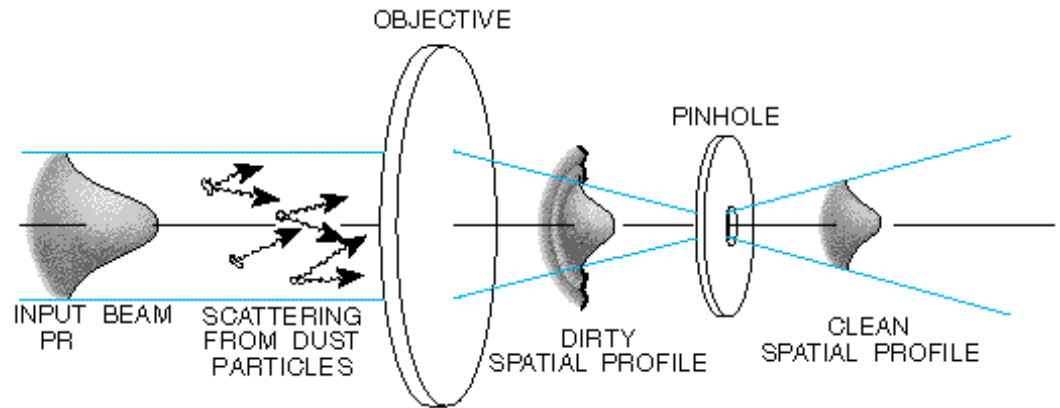


Image: demagnified source, plus aperture diffraction



Controlling spatial coherence

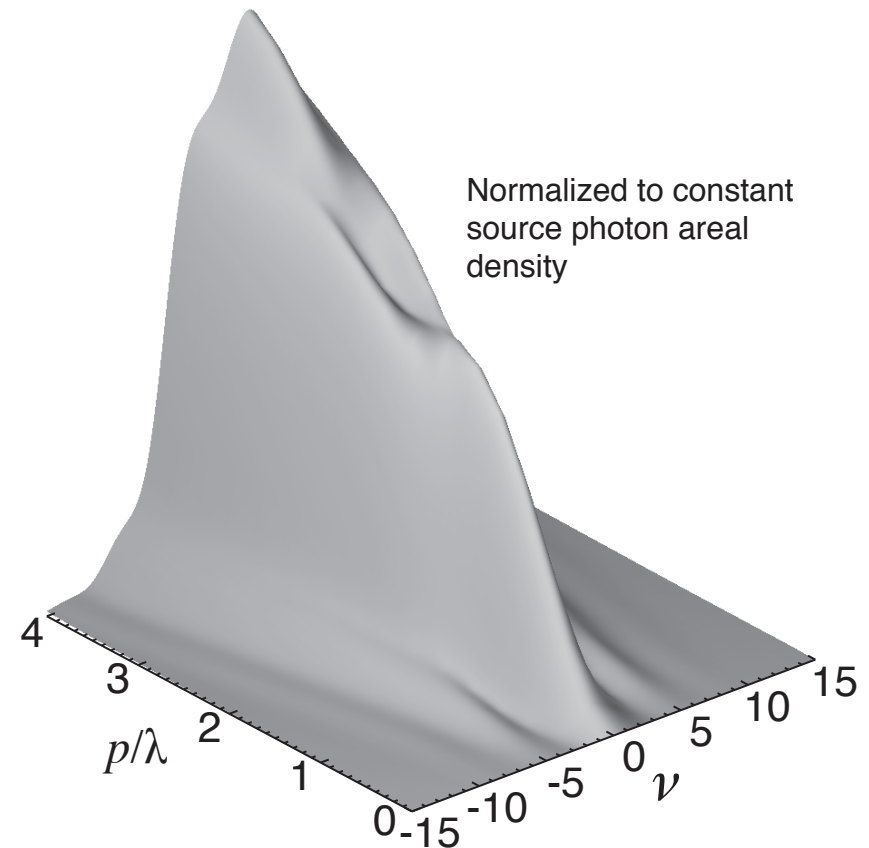
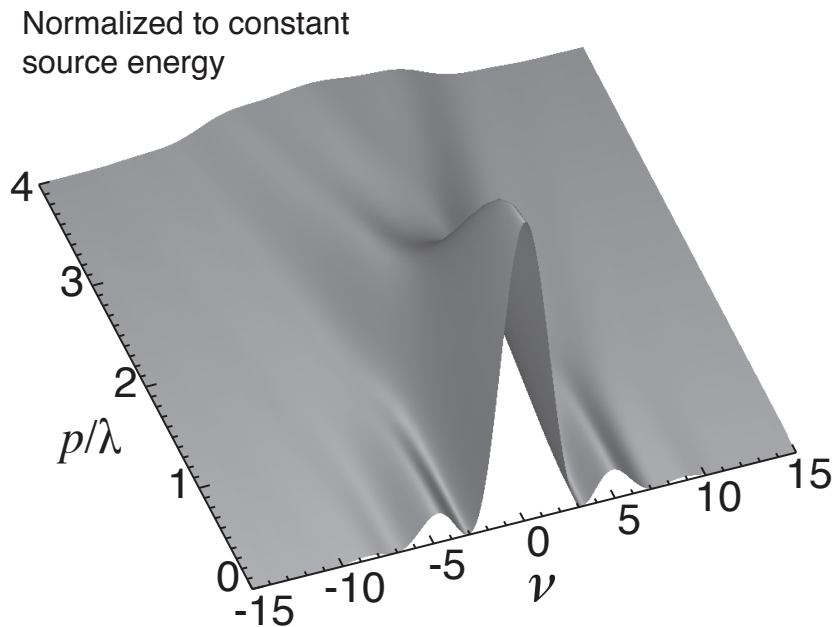
- Spatial filter: pinhole at the focus of a lens. Passes only the spatially coherent fraction of an incident beam.
- X-ray beamlines: image the source to a secondary position with an aperture, for a flux-versus-coherence tradeoff.



Diagram, photo from Newport catalog

Phase space area and probe focus

How close must $p=(\text{source diameter})\cdot(\text{optic's full subtended angle})$ be to λ ? $p \approx 1 \cdot \lambda$ works pretty well!

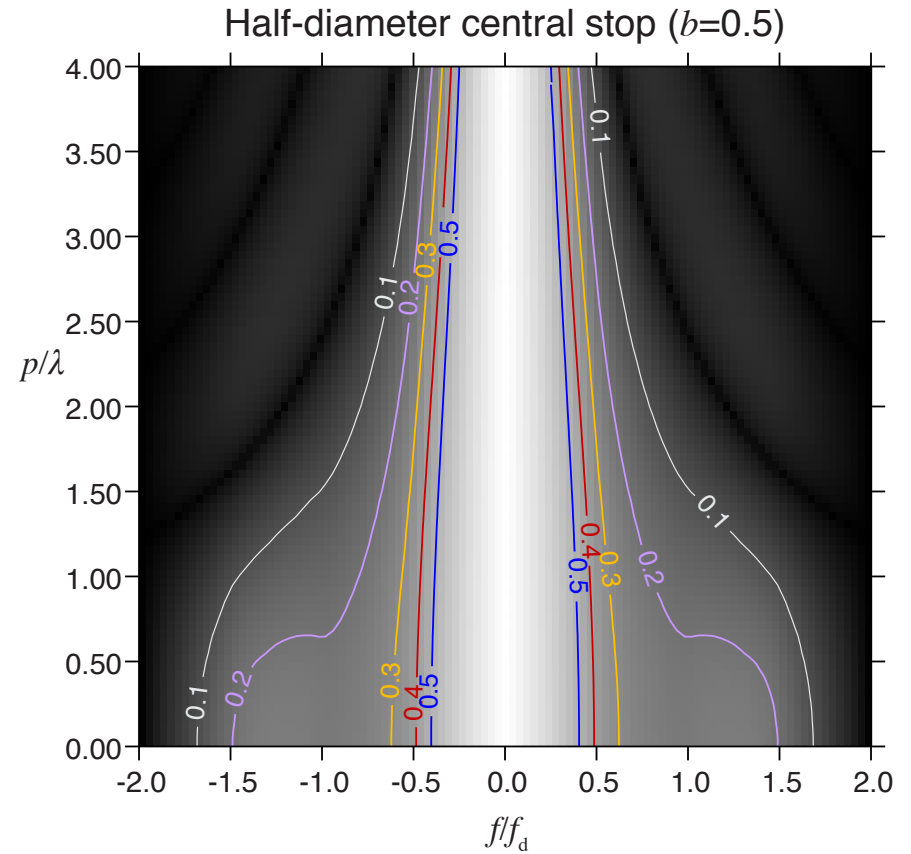
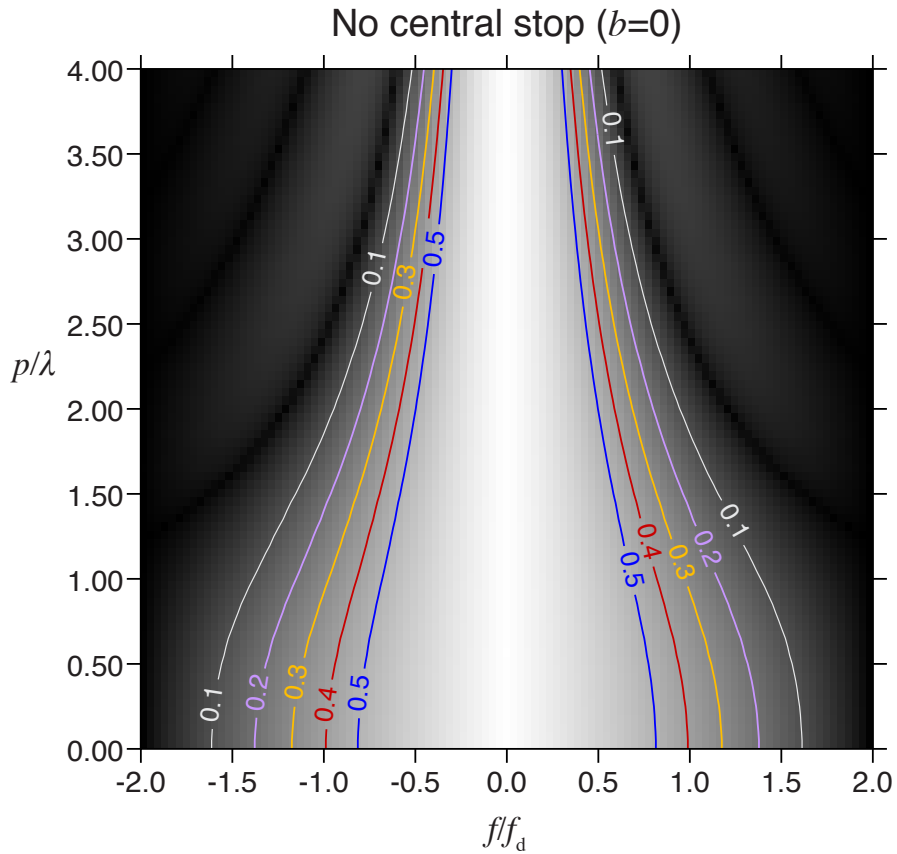


Effect on point spread function PSF (50% central stop)

Jacobsen *et al.*, *Ultramicroscopy* **47**, 55 (1992); Winn *et al.*, *J. Synch. Rad.* **7**, 395 (2000).

Transfer function view of source phase space p

- Contours of transfer function value



Coherence and nanoprobes

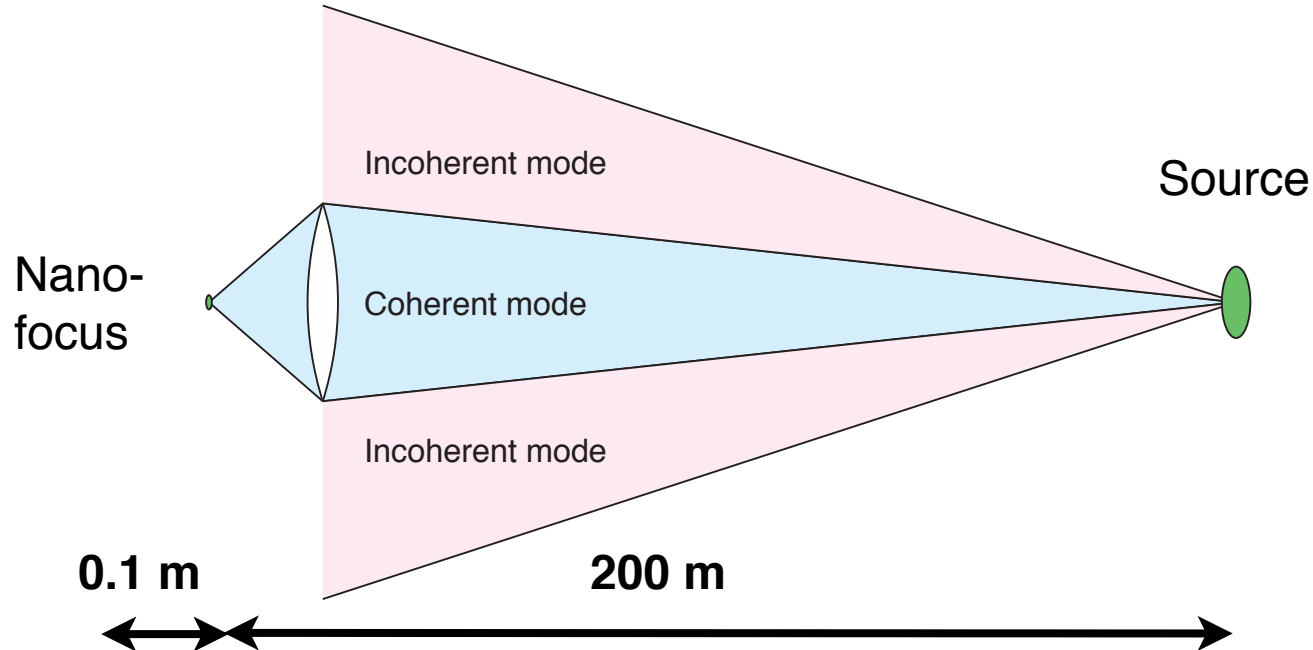
- You can only reach the diffraction limit (as opposed to the source-limited limit) of focusing from a lens when you accept a single coherent mode with phase space area $\sim 1\lambda$ in each direction.
- If the phase space of your source is larger, you must throw away the incoherent modes! Example with the APS at Argonne:

Parameter	electron beam	photon beam	MBA Upgrade combination	today
Horizontal size	$\sigma_x = 21.5 \mu\text{m}$	$\sigma_r = 5.5 \mu\text{m}$	$\Sigma_x = 22.2 \mu\text{m}$	275.0 μm
Horizontal divergence	$\sigma'_x = 3.1 \mu\text{rad}$	$\sigma'_r = 3.6 \mu\text{rad}$	$\Sigma'_x = 4.8 \mu\text{rad}$	12.1 μrad
Vertical size	$\sigma_y = 4.0 \mu\text{m}$	$\sigma_r = 5.5 \mu\text{m}$	$\Sigma_y = 6.8 \mu\text{m}$	10.7 μm
Vertical divergence	$\sigma'_y = 1.7 \mu\text{rad}$	$\sigma'_r = 3.6 \mu\text{rad}$	$\Sigma'_y = 4.0 \mu\text{rad}$	6.2 μrad
Phase space parameter p_x	$(2.35\Sigma_x)(2.35\Sigma'_x)/\lambda$		4.69 at 10 keV	148.53
Phase space parameter p_y	$(2.35\Sigma_y)(2.35\Sigma'_y)/\lambda$		1.20 at 10 keV	2.95
$p_x \cdot p_y / \lambda^2$			5.7	438.1



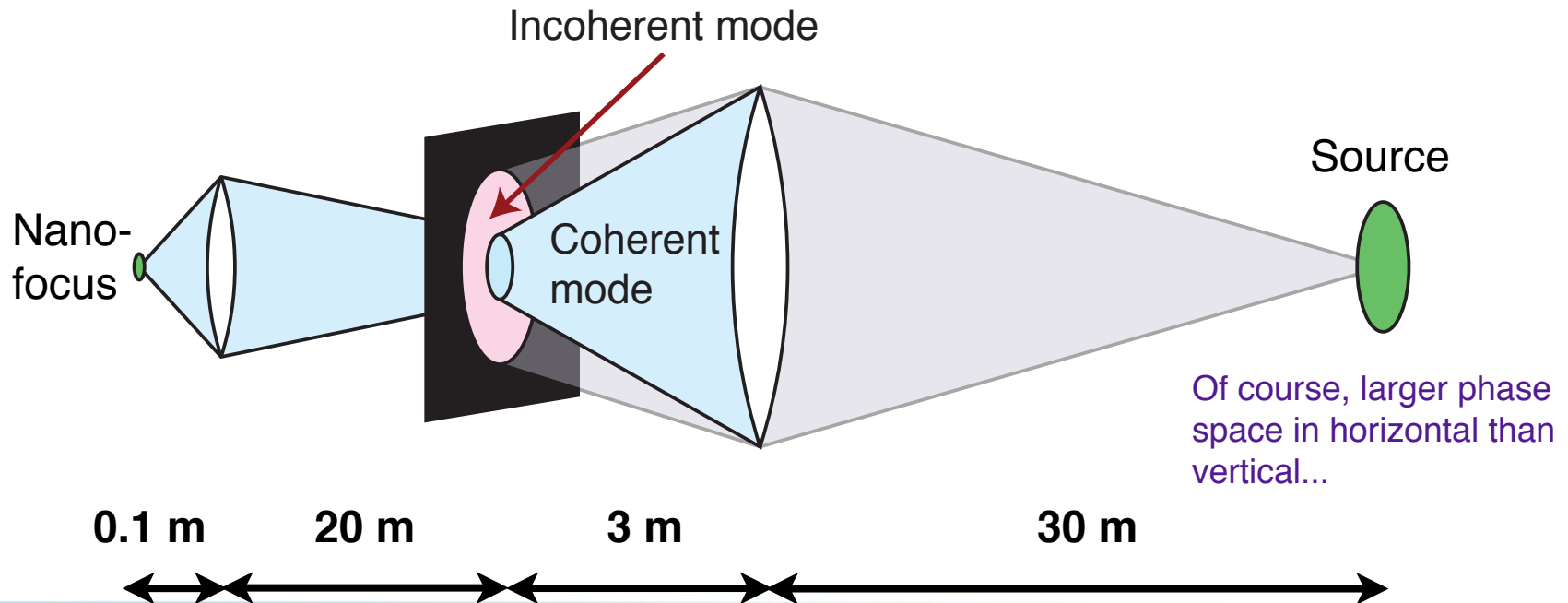
How to limit modes?

- Approach 1: distance, with a single nanofocusing optic.
 - Need to demagnify the source size in one step; for a fixed focal length (thus working distance) optic, you need a longer beamline.
 - **Example: $40\ \mu\text{m}$ to $20\ \text{nm}$ in $200/0.1$ meters. Total length: **200 m.****
 - For an optic with fixed diameter (and thus numerical aperture, and resolution), you have no flux-versus-resolution tradeoff.



How to limit modes?

- Approach 2: aperture at an intermediate focus.
 - Two-stage demagnification means you can work with a shorter beamline!
 - **Example: demagnify source size from $40\ \mu\text{m}$ to $4\ \mu\text{m}$ in 30/3 meters, then $4\ \mu\text{m}$ to 20 nm in 20/0.1 meters. Total length $30+3+20=53\ \text{m}$.**
 - You can adjust your intermediate aperture and have a flux-versus-resolution tradeoff.



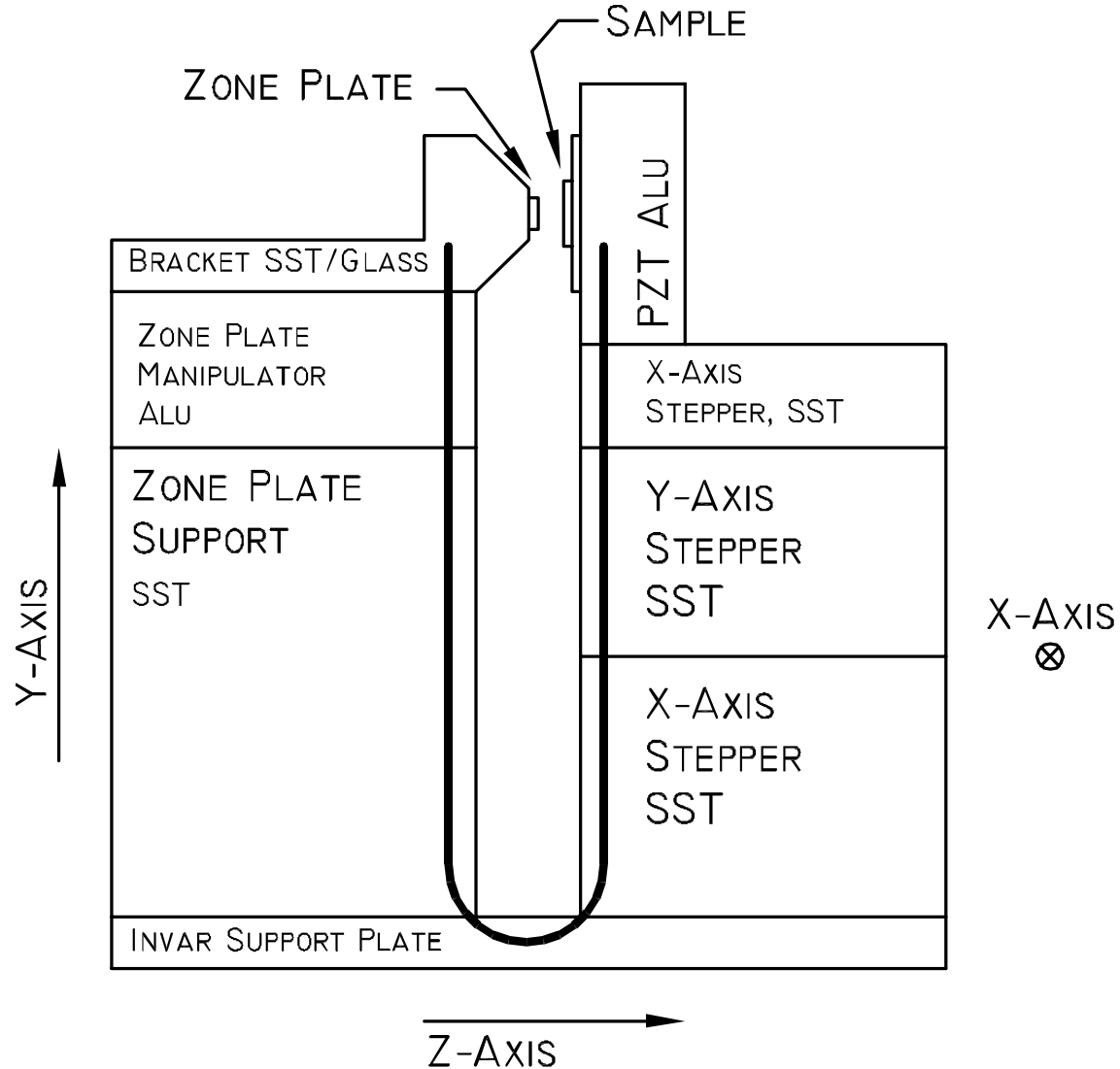
Outline

- X-ray optics and x-ray lenses
- Why scanning?
- Illumination considerations
- **Instrumentation considerations**
- Absorption spectromicroscopy
- CDI and ptychography



Scanning microscope instrumentation

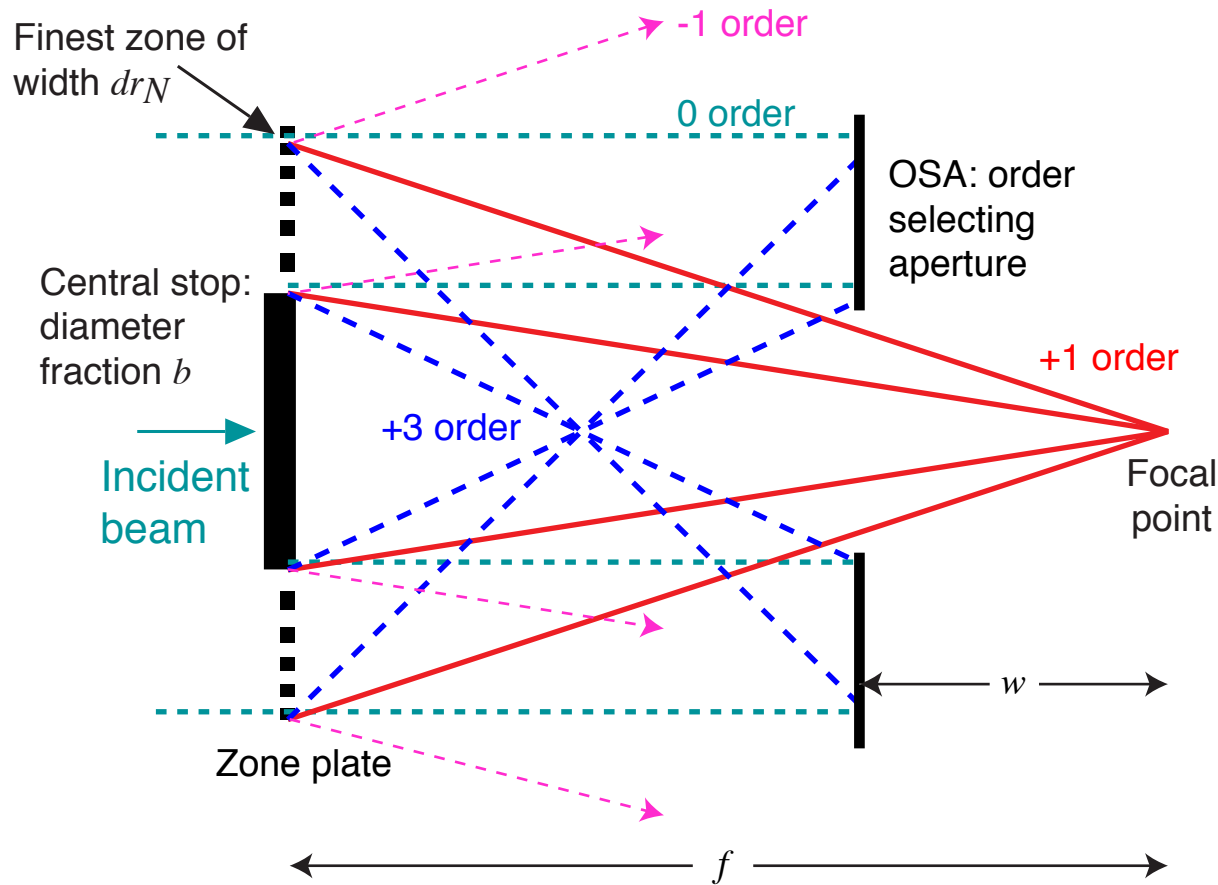
- Mechanical design principles: small, light and stiff, with resonant frequencies >100 Hz.
- Thermal expansion: many materials have $\sim 10^{-5}$ per C which is $1 \mu\text{m}$ per 10 cm.
- Stepper motors on bearings for large range scans, and positioning ($0.1 \mu\text{m}$ resolution over many mm).
- Piezoelectric pushers on flexure guides for fine scans (10 nm resolution over $20\text{-}100 \mu\text{m}$).



M. Feser PhD dissertation: tinyurl.com/kowp6vs

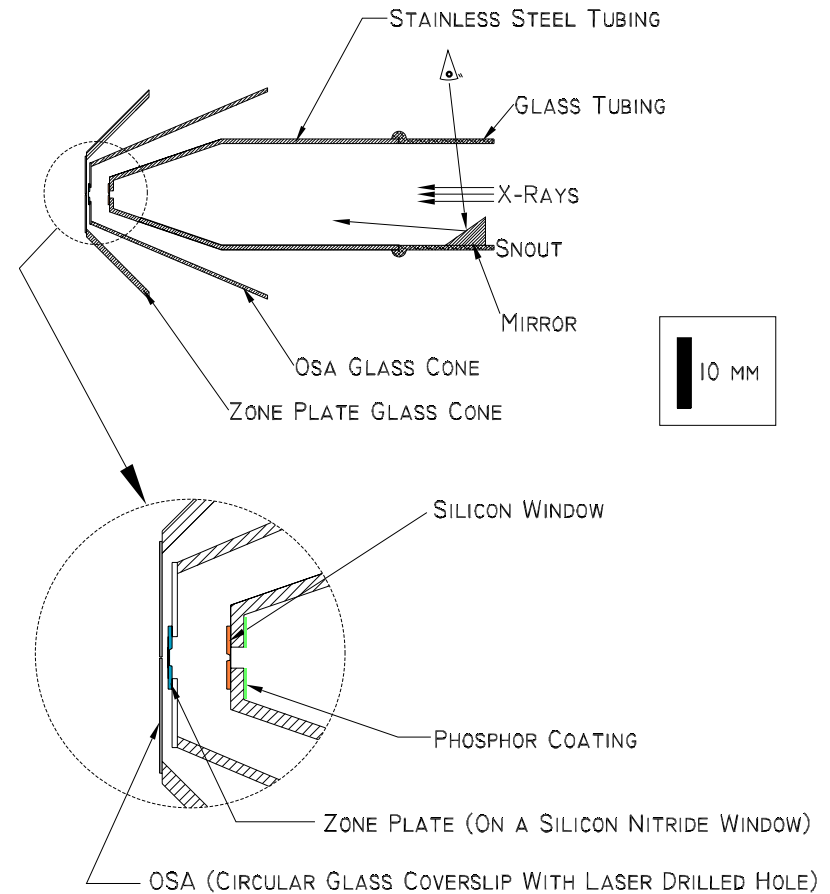
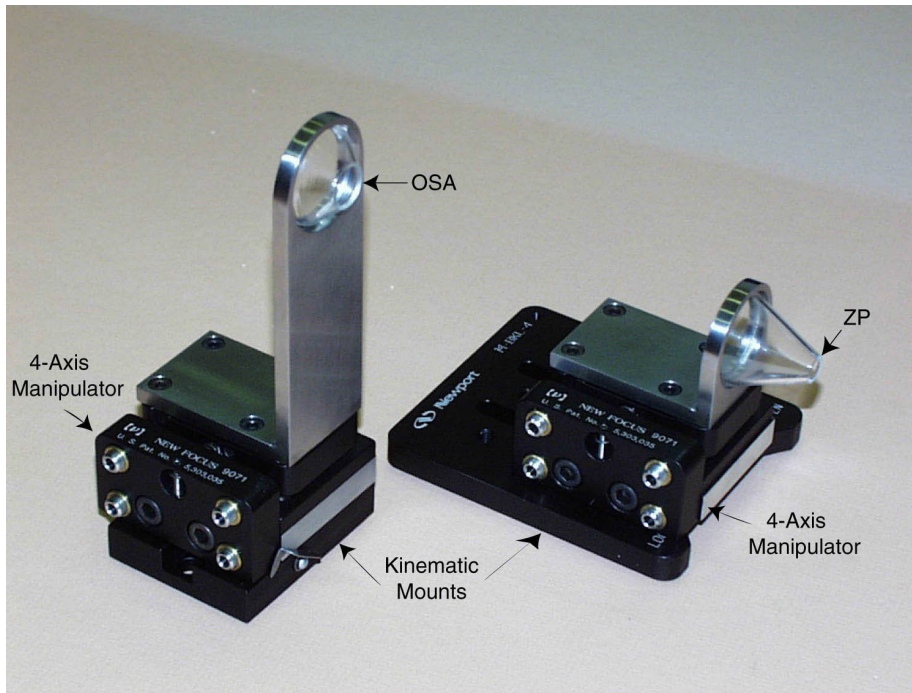
Zone plates and diffraction orders

- Central stop fraction b is 0.5 for soft x-rays, and less with hard x-rays (longer focal length so more generous with working distance w)



Swappable zone plates and paired order selecting apertures (OSAs)

- Glass cones: see-through, low thermal expansion coefficient
- Low-cost (commercial) kinematic mounts and position/tilt adjusters



M. Feser PhD dissertation: tinyurl.com/kowp6vs



Outline

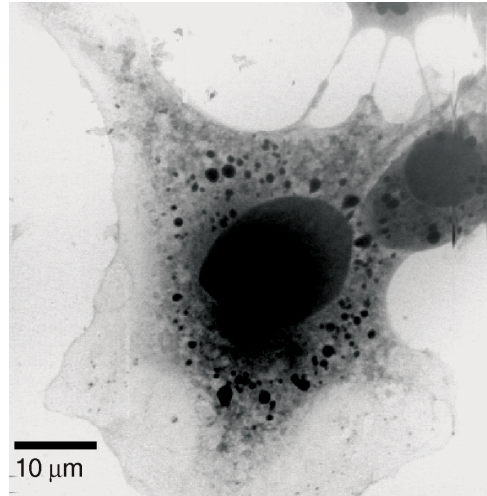
- X-ray optics and x-ray lenses
- Why scanning?
- Illumination considerations
- Instrumentation considerations
- **Absorption spectromicroscopy**
- Fluorescence microscopy
- CDI and ptychography



2D imaging with Stony Brook STXMs

2D imaging is moderately useful but...

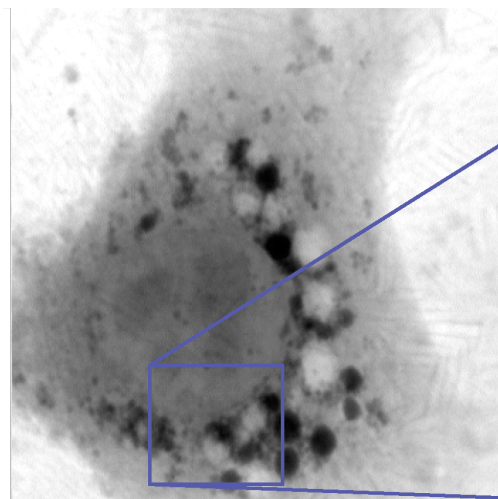
- Cannot track fluorescently-labeled proteins in living cells
- Resolution is inferior to cryoEM, though do not need to section
- Best utility may lie beyond simple 2D imaging



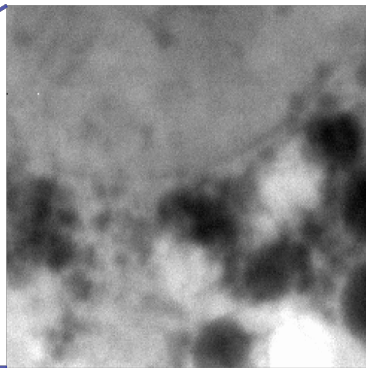
NIL 8 fibroblast (wet, fixed): Oehler *et al.*



Human sperm (unfixed): Wirick, Fleckenstein, Sheynkin *et al.*



7 μm



2 μm

Fibroblast (frozen hydrated): Maser *et al.*, *J. Microsc.* **197**, 68 (2000)



Energy scales

- Chemistry: C-H bond is 104 kcal/mol or 435 kJoules/mol:

$$\left(435 \frac{\text{kJ}}{\text{mol}}\right) \cdot \left(\frac{10^3 \text{ J}}{\text{kJ}}\right) \cdot \left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}\right) \cdot \left(\frac{1}{6.02 \times 10^{23} \text{ molecules/mol}}\right)$$

or $E=4.5$ eV/molecule.

- Photon wavelength associated with 4.5 eV is

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$$

or $\lambda=275$ nm.

- More broadly, chemical bonds involve 300-900 kJ/mol, or 3-9 eV/molecule, or 130-400 nm light.
- Bohr model:

$$E = -E_0 \frac{Z^2}{n^2} = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

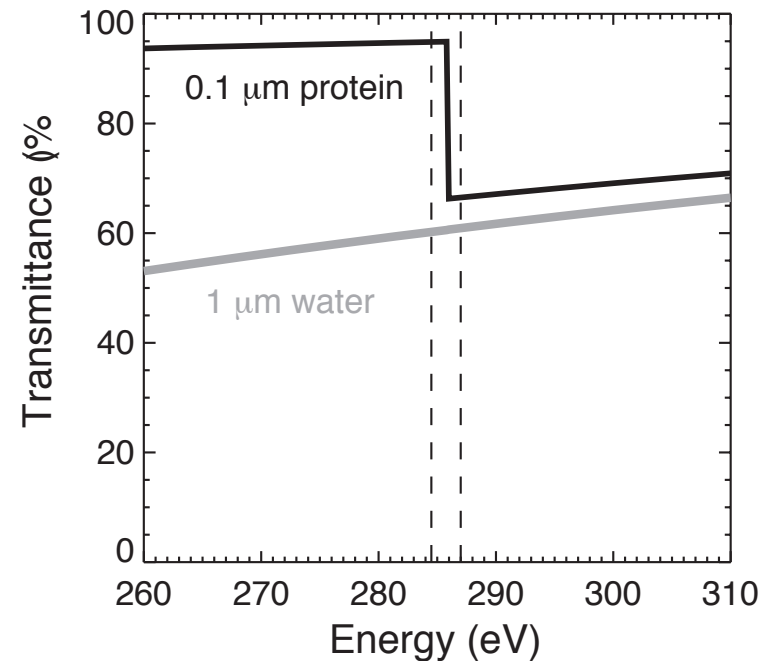
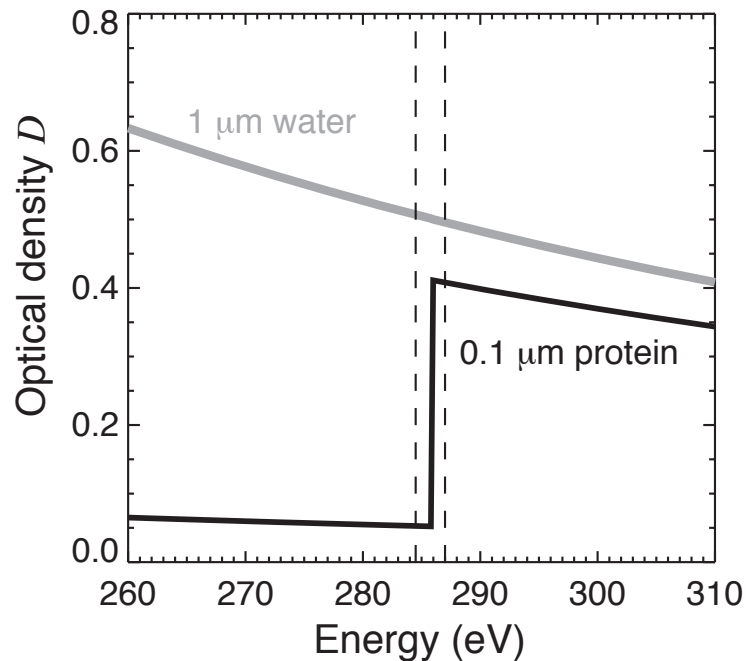
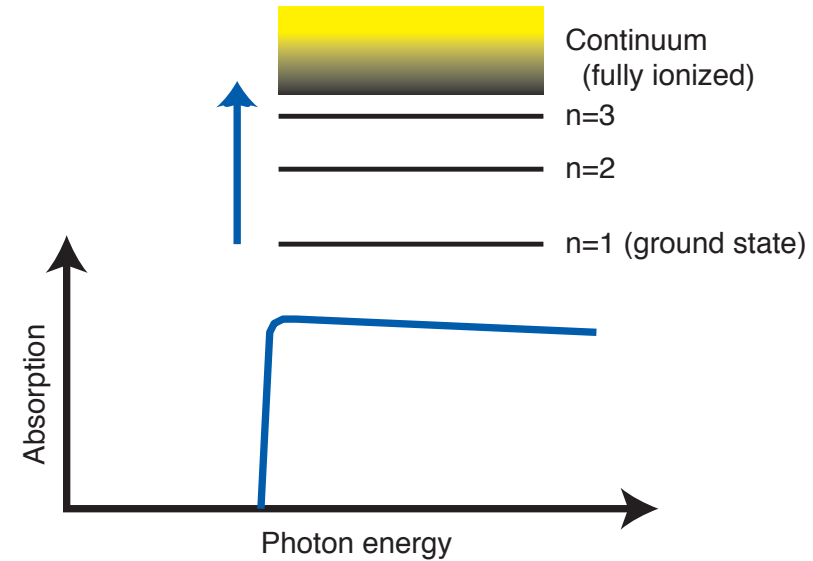


Absorption edges

Lambert-Beer law: linear absorption coefficient μ

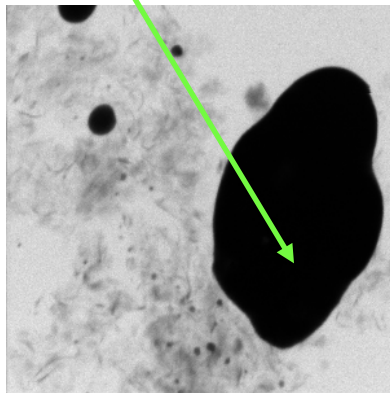
$$I = I_0 e^{-\mu(E) \cdot t} = I_0 e^{-D(E)}$$

This coefficient makes a jump at specific elemental absorption edges!
This example: 0.1 μm protein in water



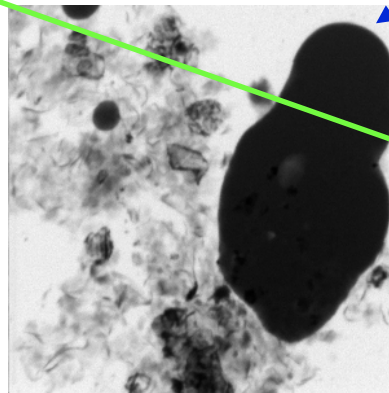
X-ray microscopy of colloids

- U. Neuhäusler (Stony Brook/Göttingen), S. Abend (Kiel), G. Lagaly (Kiel), C. Jacobsen (Stony Brook), *Colloid and Polymer Science* **277**, 719 (1999).
- Emulsion: water, oil droplets, clay, and layered double hydroxides (LDH).
- “Caged” part of oil droplet remains fixed; “uncaged” part can disperse.



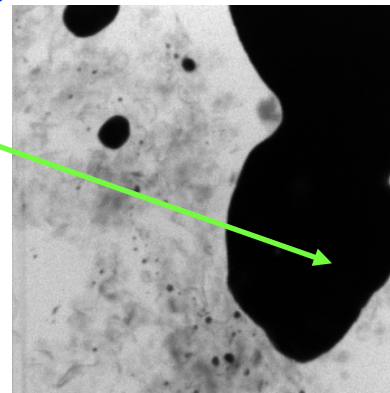
4 microns

346 eV: calcium weakly absorbing. Clays and LDHs absorb equally



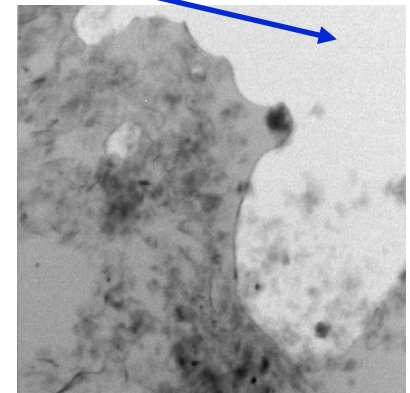
4 microns

352.3 eV: calcium strongly absorbing. Calcium-rich LDHs are highlighted.



4 microns

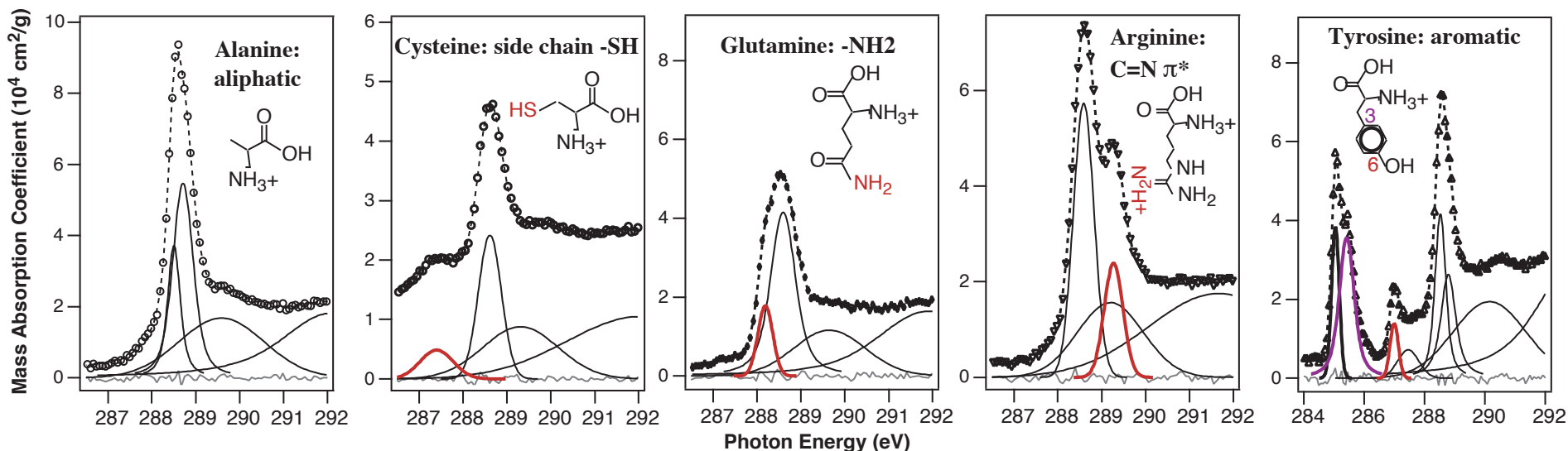
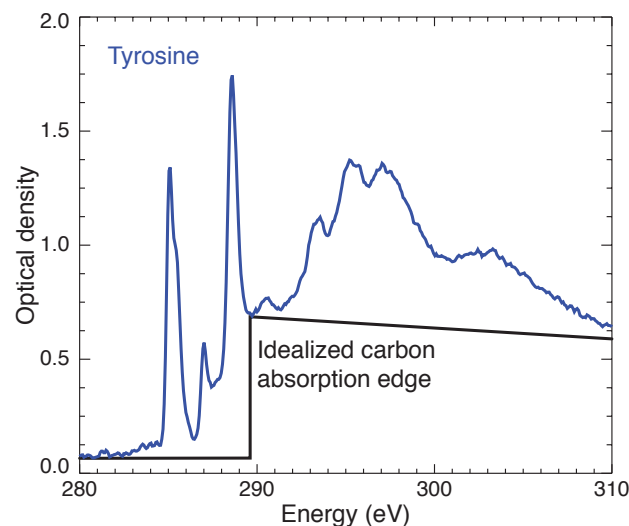
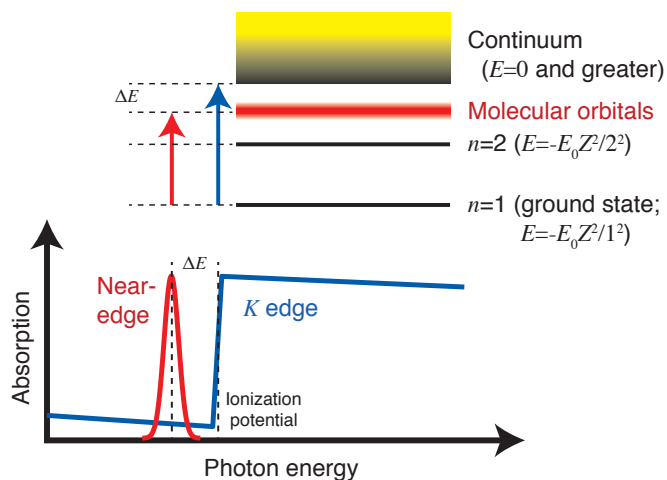
290 eV: carbon strongly absorbing



4 microns

284 eV: carbon (oil drop) weakly absorbing

Near-edge absorption fine structure (NEXAFS) or X-ray absorption near-edge structure (XANES)



Amino acids example: K. Kaznatcheyev *et al.*, *J. Phys. Chem. A* **106**, 3153 (2002)

Near-edge spectroscopy: ELNES and XANES

ELNES (electron Energy Loss)

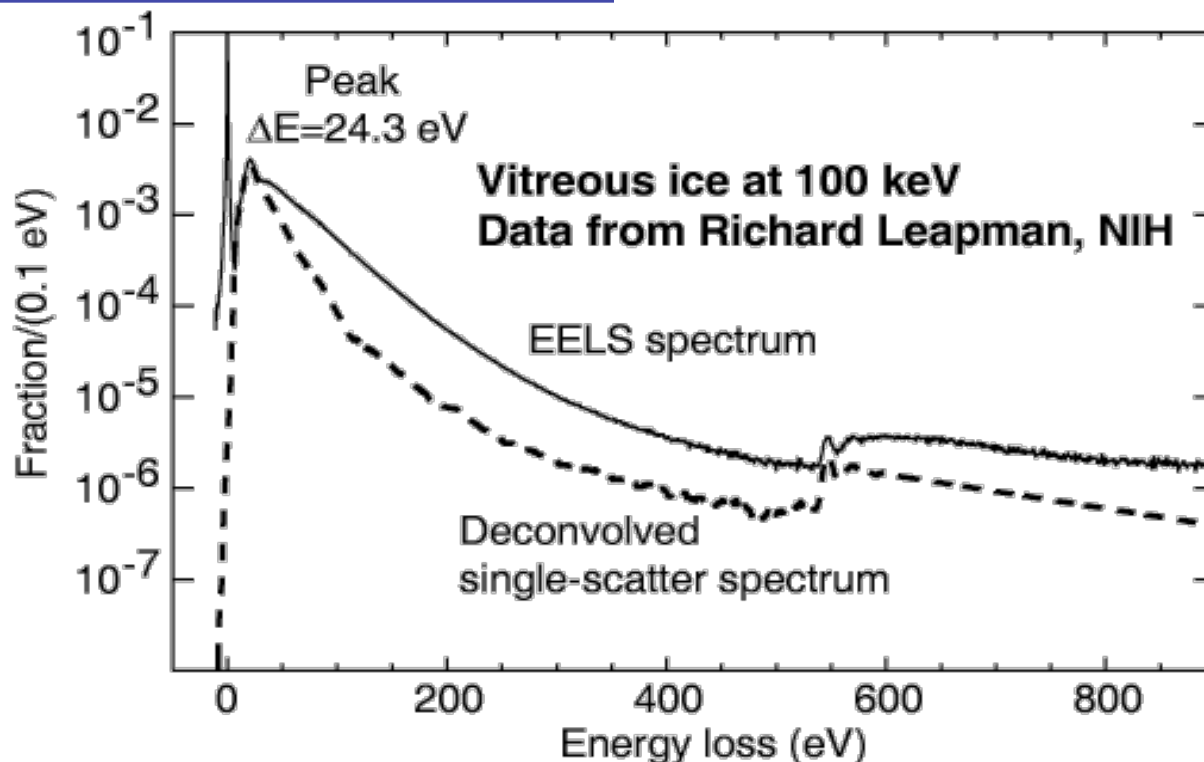
- Plural inelastic scattering
- Many elements at once - but plasmon modes are always excited (damage)
- ΔE was ~ 0.6 eV, but now < 0.1 eV in some cases

XANES (X-ray Absorption)

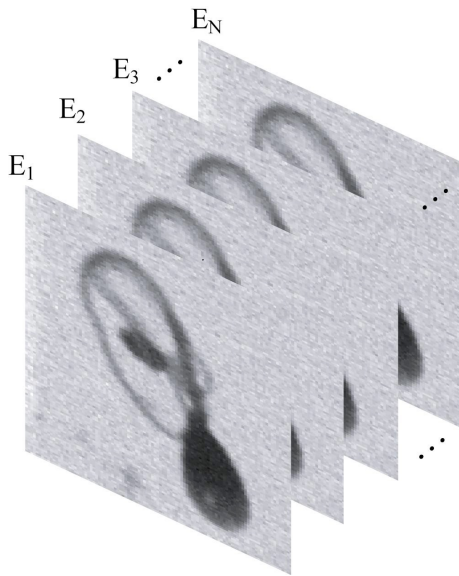
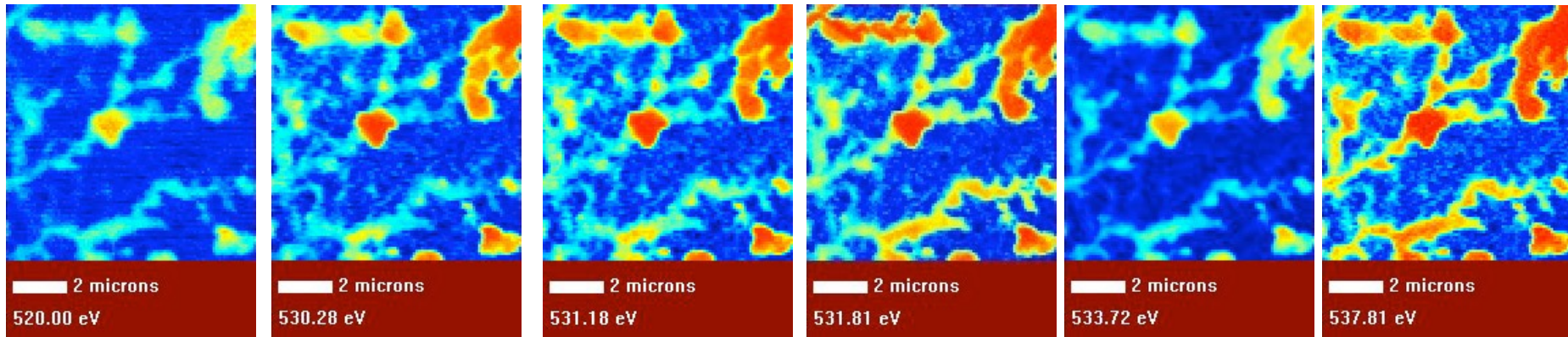
- No plural scattering
- One element at a time - slow but less damage
- ΔE of 0.05-0.1 eV is common

Electrons $\sim 1000x$ more damaging:

- Isaacson and Utlaut, *Optik* **50**, 213 (1978).
- Rightor *et al.*, *J. Phys. Chem. B* **101**, 1950 (1997).



Spectromicroscopy from image sequences



Lu in hematite (T. Schäfer)

10^4 - 10^5 spectra! Too many to analyze “by hand.”
Complex mixtures *etc.*; life is not made up of uniform thin films.

How to deal with the complexity?

1. PCA for noise reduction, orthogonalization.
2. Cluster analysis to find patterns.
3. Non-negative matrix factorization for final analysis.

Aligned spectral image sequences:
Jacobsen *et al.*, *J. Microscopy* **197**,
173 (2000)

Optimization techniques in spectromicroscopy analysis

Global cost function F :

$$F(\mu, t) = \|D - \mu \cdot t\|_2^2 + \lambda_{\text{sparse}} \|t\|_1 + \lambda_{\text{sim}} \|\mu - \mu_{\text{cluster}}\|_2^2$$

- **Data matching**: reconstruction from spectra μ and images t should match optical density data D .
- **Sparseness**: maximize the separation of features in the images t . The L_1 norm provides a good measure of sparseness.
- **Similarity**: keep the spectra μ similar to those found in regions of the sample (μ_{cluster}).
- The regularization parameters λ are like a currency exchange rate to put all the costs on the same scale; increase λ until data matching error begins to grow significantly.
- Partial derivatives of each cost guide iterative solutions.
- Mak, M. Lerotic, H. Fleckenstein, S. Vogt, S. Wild, S. Leyffer, Y. Sheynkin, and C. Jacobsen, *Faraday Discussions* **171**, 357 (2014).

Optimized analysis

Faraday Discussions

Cite this: DOI: 10.1039/c4fd00023d

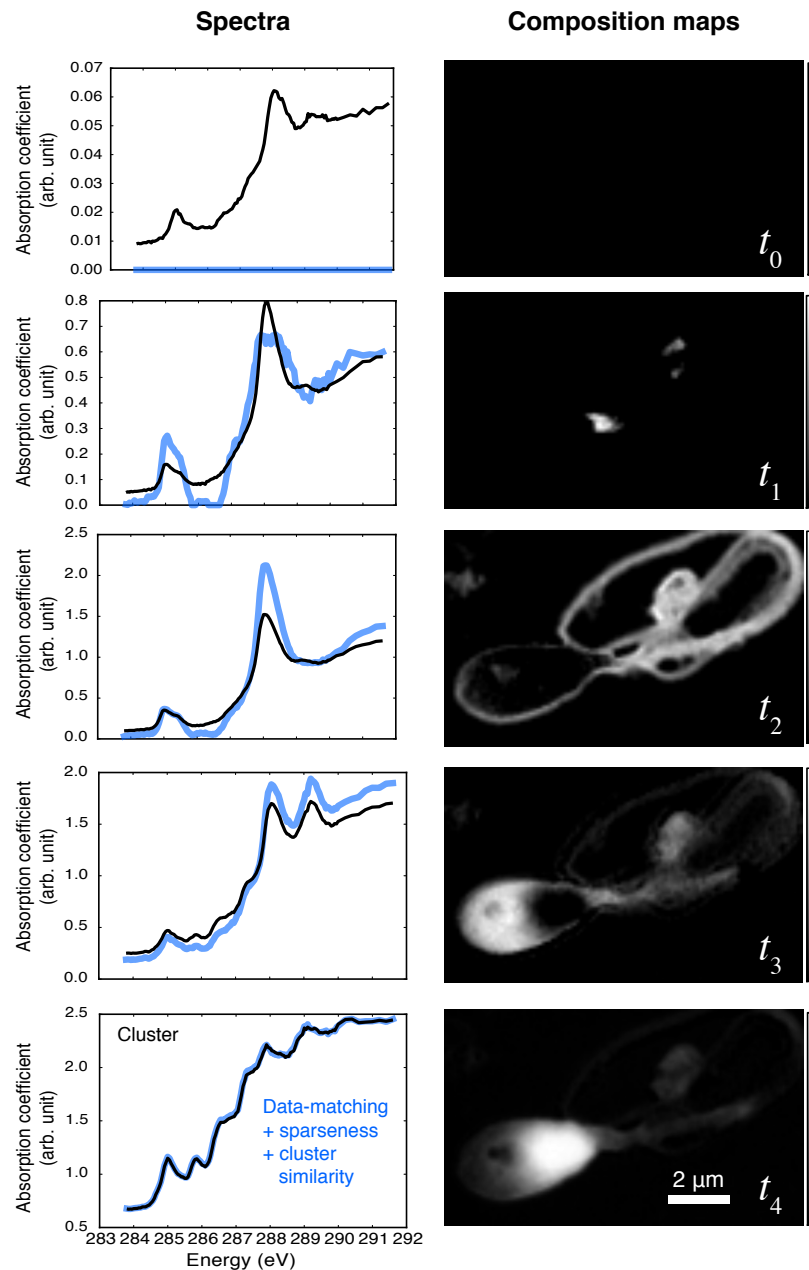
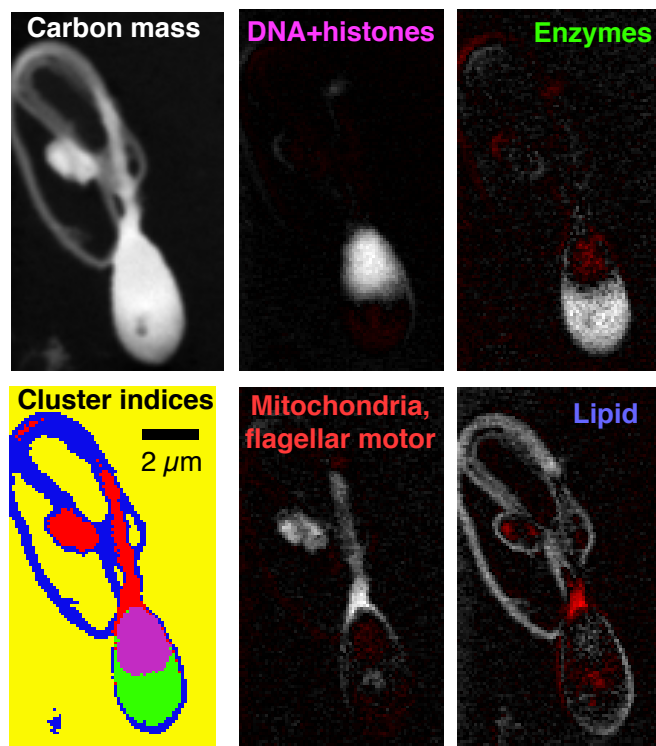


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Non-negative matrix analysis for effective feature extraction in X-ray spectromicroscopy

Rachel Mak,^a Mirna Lerotic,^b Holger Fleckenstein,^c Stefan Vogt,^d Stefan M. Wild,^e Sven Leyffer,^e Yefim Sheynkin^f and Chris Jacobsen^{*dag}



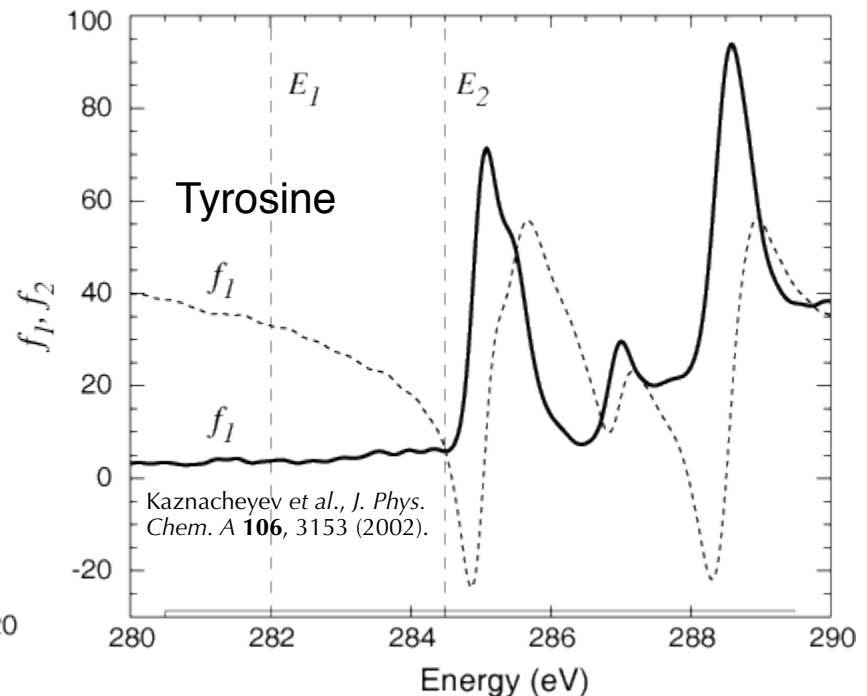
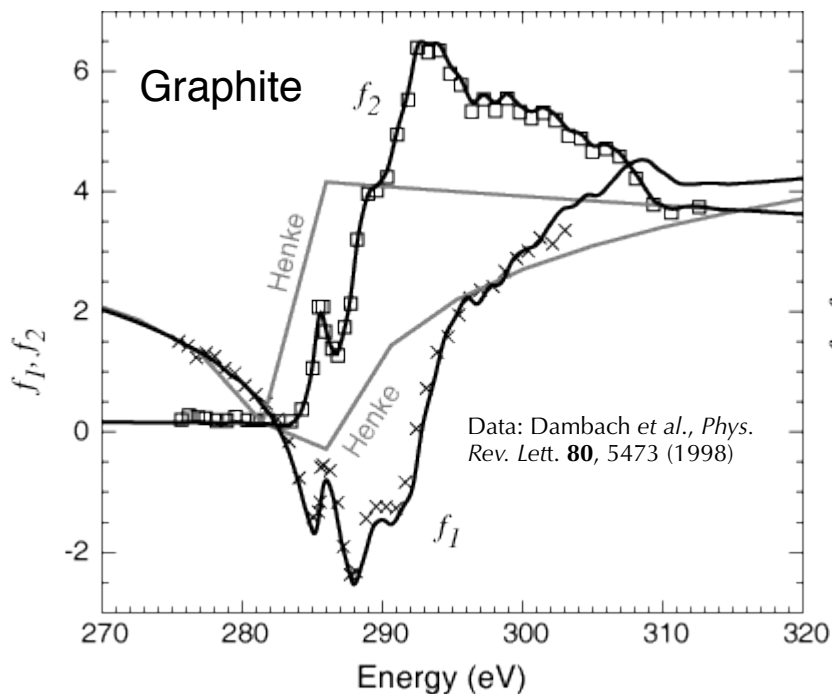
Phase contrast spectromicroscopy?

Kramers-Kronig transform:

phase shift $f_1(E)$ from absorption $f_2(E)$

$$f_1(E) = Z + \frac{2}{\pi} \int_0^{\infty} \frac{\epsilon f_2(\epsilon)}{E^2 - \epsilon^2} d\epsilon - \Delta f_r,$$

Requires phase contrast imaging, such as in ptychography



See Jacobsen et al, SPIE **5538**, 23 (2004); Maiden et al., Nature Comm. **4**, 1669 (2013).