



A. Kaestner :: Paul Scherrer Institut

Introduction to computed tomography

Theory and practical details for the experimentalist

- 1 Introduction
- 2 Reconstruction
- 3 Beam geometry
- 4 Sampling
- 5 Artefacts
- 6 Summary

Introduction

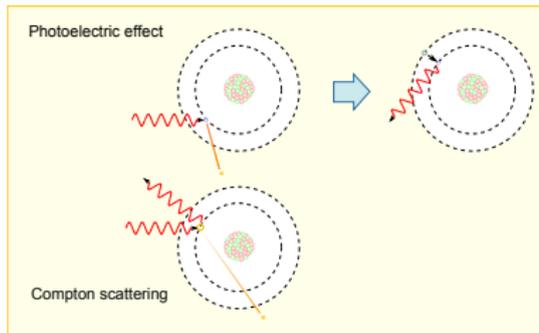
- Understand the image formation process
- Understand the differences between analytical and iterative reconstruction
- Knowing key parameters in tomographic reconstruction and how they impact the resulting images
- Recognizing typical artifacts and how to remove them

We have a solid item to investigate. . .

- For a first look of the outside
- Cut the item in pieces
- Next step, use a transmission image

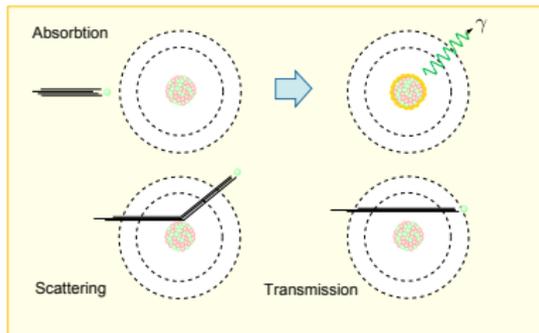


X-rays



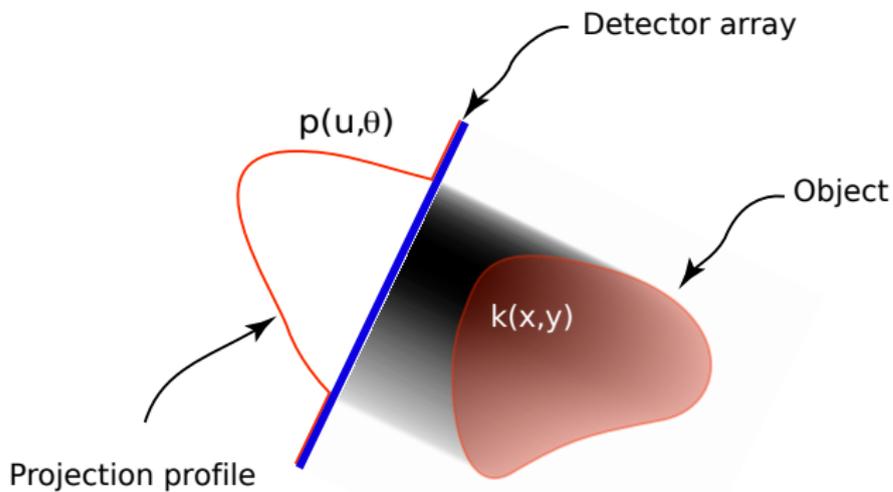
- Electromagnetic radiation.
- Interaction with the electron shells.

Neutrons



- Neutral particle beam.
- Interaction with the nucleus.

A ray illuminates a semi-transparent medium



A ray penetrating a medium is attenuated according to Beer-Lamberts law The intensity is attenuated in the medium according to

$$I = I_0 e^{-\int_L k(x,y) dl}$$

I - Intensity behind the sample

I_0 - Incident intensity

k - Attenuation coefficient,

μ - Linear attenuation coefficient X-rays

Σ - Macroscopic cross-section for neutrons

L - Line through the sample.

Computing an attenuation image

From Beer-Lamberts law we get

$$p = -\log \left(\frac{r - r_{DC}}{r_{OB} - r_{DC}} \right) = -\log \left(\frac{\begin{array}{c} \text{[Image: Measured radiogram]} \\ \text{---} \\ \text{[Image: Dark current image]} \end{array}}{\begin{array}{c} \text{[Image: Open beam image]} \\ \text{---} \\ \text{[Image: Dark current image]} \end{array}} \right) = \text{[Image: Normed projection]}$$

p Normed projection

r Measured radiogram

r_{DC} Dark current image (removes noise floor)

r_{OB} Open beam image, measured I_0

Each pixel represent the line integral $\int_L k(x)dx$ through the sample.

Piecewise constant sample

Few discrete regions

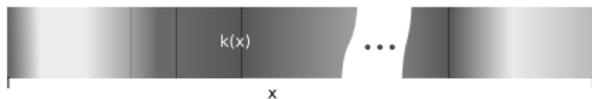
$$I = I_0 e^{-\sum_{i=1}^N k_i x_i}$$



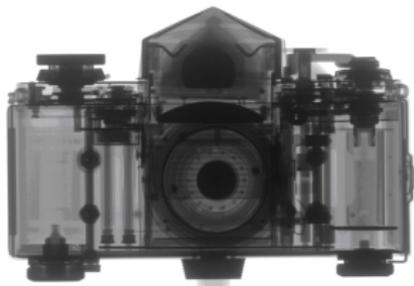
Continuous samples

Let $x_i = \Delta x$ and $\Delta x \rightarrow 0$

$$I = I_0 e^{-\int_L k(x) dx}$$

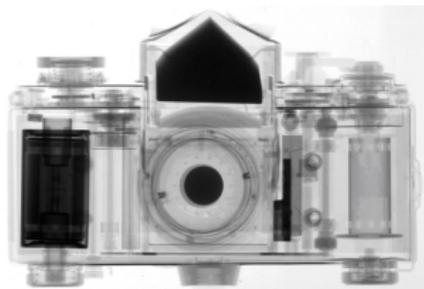


X-rays at 150keV



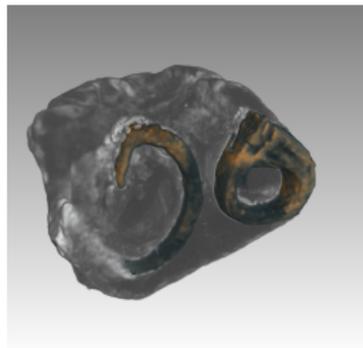
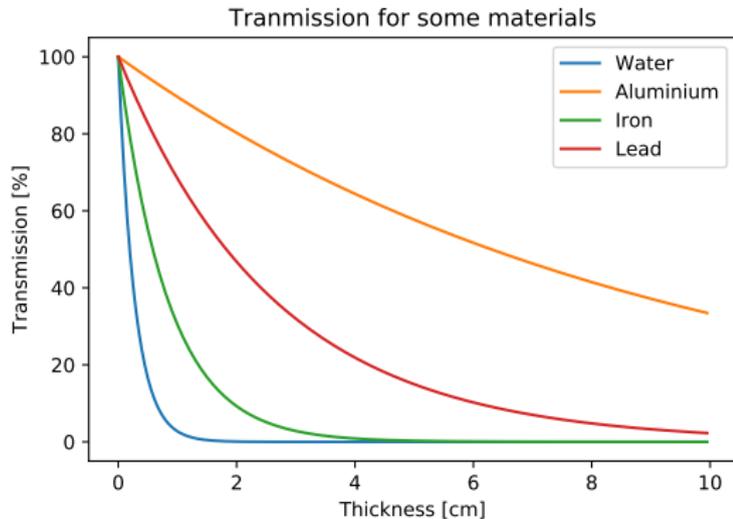
Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 1.02																	He 0.02
2	Li 0.08	Be 0.22											B 0.28	C 0.27	N 0.11	O 0.16	F 0.14	Ne 0.17
3	Na 0.13	Mg 0.24											Al 0.38	Si 0.33	P 0.25	S 0.30	Cl 0.23	Ar 0.15
4	K 0.14	Ca 0.28	Sc 0.48	Ti 0.73	V 1.04	Cr 1.29	Mn 1.52	Fe 1.57	Ni 1.78	Cu 1.98	Zn 1.87	Ga 1.66	Ge 1.45	As 1.33	Se 1.28	Br 1.23	Kr 0.90	Rb 0.73
5	Rb 0.47	Sr 0.86	Y 1.61	Zr 2.47	Nb 3.43	Mo 4.29	Tc 5.59	Ru 5.71	Rh 6.15	Pd 5.67	Cd 4.84	In 4.31	Sn 3.98	Sb 4.23	Te 4.65	I 3.45	Xe 2.53	
6	Cs 1.47	Ba 2.73		Hf 19.76	Ta 25.47	W 35.48	Re 34.47	Os 27.80	Ir 30.01	Pt 30.61	Au 35.94	Hg 25.88	Tl 22.23	Pb 25.81	Bi 20.28	Po 20.52	At 18.71	Rn 17.77
7	Fr 11.68	Ra 11.68	Rf -	Db -	Sg -	Bh -	Hs -	Mt -	Ds -	Rg -	Uub -	Uuq -	Uup -	Uuh -	Uus -	Uuo -		
Lanthanides																		
	La 5.04	Ce 2.79	Pr 4.25	Nd 6.45	Pm 7.33	Sm 7.68	Eu 5.89	Gd 6.69	Tb 9.41	Dy 10.17	Ho 10.17	Er 11.30	Tm 12.40	Yb 9.32	Lu 14.07			
Actinides																		
	Ac 24.47	Th 38.95	Pa 39.62	U 49.68	Np -	Pu -	Am -	Cm -	Bk -	Cf -	Es -	Fm -	Md -	No -	Lr -			

Thermal neutrons



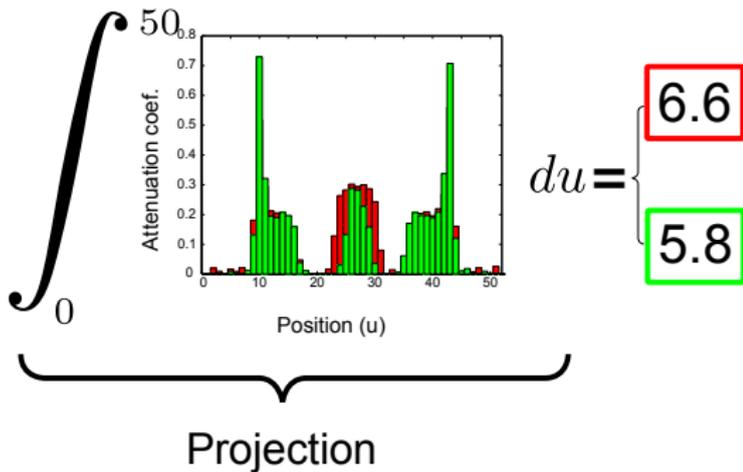
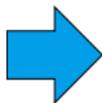
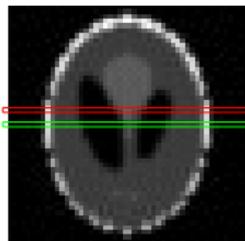
Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 3.34																	He 0.02
2	Li 0.30	Be 0.75																Ne 0.10
3	Na 0.35	Mg 0.35																Ar 0.53
4	K 0.06	Ca 0.08	Sc 2.02	Ti 0.60	V 0.62	Cr 0.72	Mn 0.94	Fe 1.21	Ni 1.19	Cu 1.02	Zn 2.05	Cd 1.07	Zr 0.35	Sr 0.48	Mo 0.47	Se 0.67	Br 0.73	Kr 0.81
5	Rb 0.08	Sr 0.14	Y 0.27	Zr 0.29	Nb 0.40	Mo 0.52	Tc 1.76	Ru 0.58	Rh 16.48	Pd 0.76	Ag 4.04	Cd 1.11	Sn 1.08	Sb 0.21	Te 0.30	I 0.25	Xe 0.43	
6	Cs 0.29	Ba 0.07		Hf 0.99	Ta 1.68	W 1.63	Re 0.64	Os 3.44	Ir 0.24	Pt 16.31	Au 16.21	Hg 0.47	Tl 0.38	Pb 0.27	Bi 0.27	Po -	At -	Rn -
7	Fr -	Ra 0.38	Rf -	Db -	Sg -	Bh -	Hs -	Mt -	Ds -	Rg -	Uub -	Uuq -	Uup -	Uuh -	Uus -	Uuo -		
Lanthanides																		
	La 0.52	Ce 0.14	Pr 0.41	Nd 1.83	Pm 1.72	Sm 14.58	Eu 171.47	Gd 147.83	Tb 10.83	Dy 24.42	Ho 2.25	Er 5.48	Tm 3.53	Yb 1.46	Lu 2.75			
Actinides																		
	Ac -	Th 0.29	Pa 0.46	U 0.82	Np 9.81	Pu 15.20	Am 2.98	Cm -	Bk -	Cf -	Es -	Fm -	Md -	No -	Lr -			

Some attenuation examples for neutrons



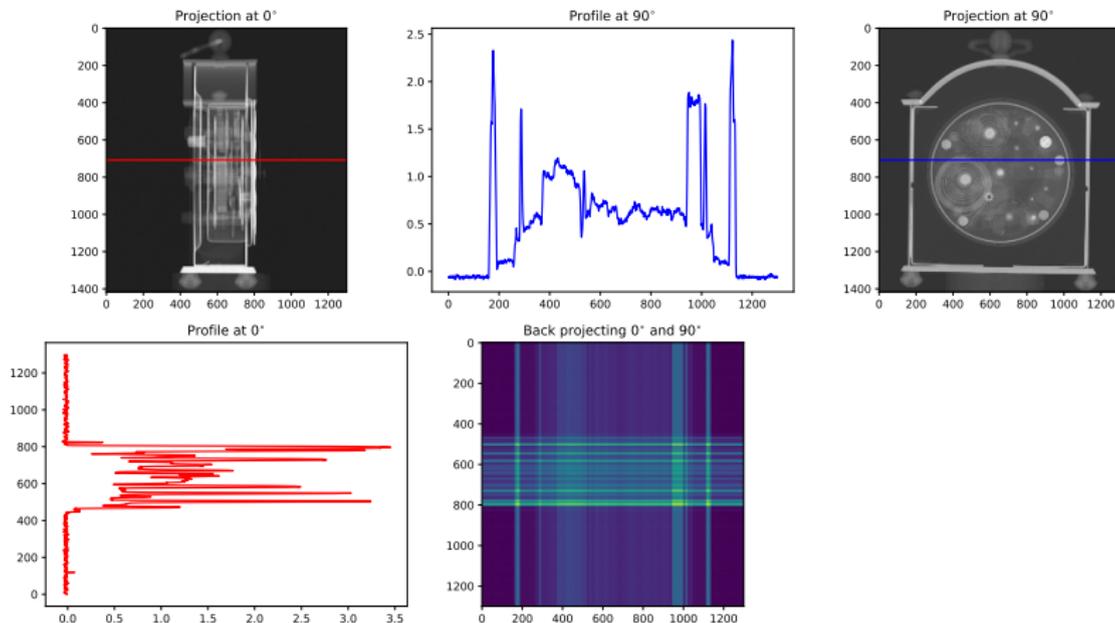
Neutron tomography of fist-sized lead canon ball from the battle of Bosworth (1485AD)

Limitation of the radiography



- Great local changes buried in the sum of bulk
- Depth position can't be determined

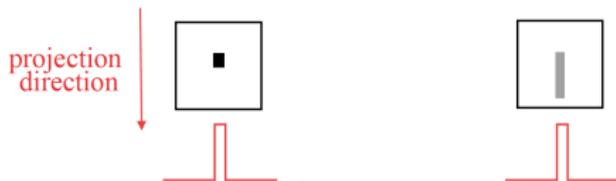
Use two projections at 90° to get depth information



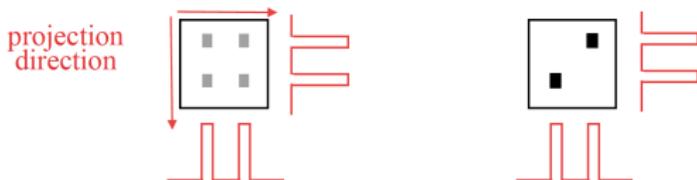
- Provides some depth information
- Still a lot of guessing

The solution is not unique

Single projection → several solutions



Two projections → several solutions



A unique solution would exist only for an infinite number of noiseless continuous projections

What is tomography?

- A method to capture three-dimensional images.
- An indirect method using projections (radiograms) to reconstruct the inner structure of a sample.
- Free translation is slice imaging
from Greek:

Tomos – 'a section' or 'a cutting'

Graph – write

- 1917 **Radon** developed the foundation for the inversion required by tomography.
- 1956 **Bracewell** the relationships between Fourier transform and Radon transform.
- 1963 First applications to medical tomography.
Kuhl obtained first backprojection.
Cormack applied Radon's results to radiograms.
- 1970 Publication of the first CT image.
- 1970-1973 **Cormack & Hounsfield** first CT scanner.
- 1979 **Cormack & Hounsfield** the Nobel prize in Medicine.



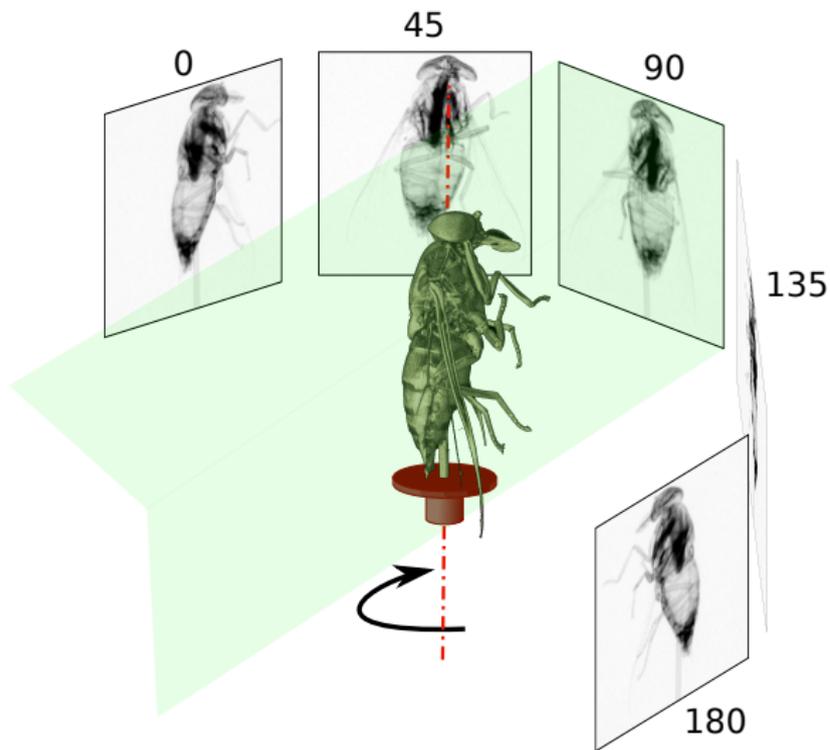
J. Radon (1887–1956)

R. Bracewell
(1921–2007)

D Kuhn (1929–2017)

A. Cormack
(1924–1998)Sir G.N. Hounsfield
(1919–2004)

Inspecting the sample from different views



Observations

$$\begin{array}{ccc} 2 & 3 & \rightarrow 5 \\ 1 & 4 & \rightarrow 5 \\ \downarrow & \downarrow & \\ 3 & 7 & \end{array}$$

Equation system

$$a_{11}x_1 + a_{12}x_2 = y_1$$

$$a_{21}x_3 + a_{22}x_4 = y_2$$

$$a_{11}x_1 + a_{21}x_3 = y_3$$

$$a_{12}x_2 + a_{22}x_4 = y_4$$

$$\vdots$$

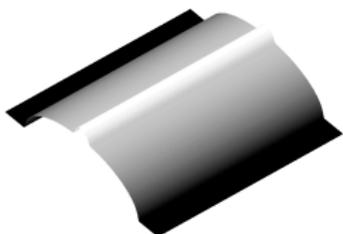
$$\Rightarrow Ax = y$$

Solve the equation system for x

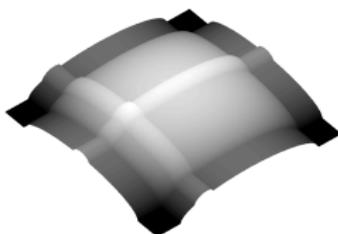
Many equations, sparse matrix A , no unique solution...

A first attempt to reconstruction: Back-projection

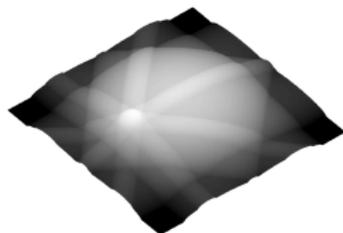
1 projections



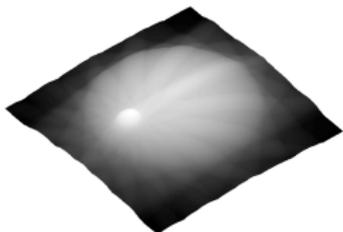
2 projections



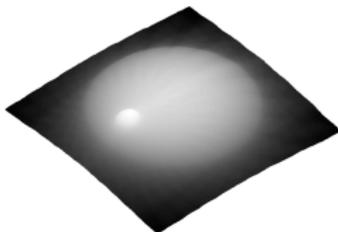
4 projections



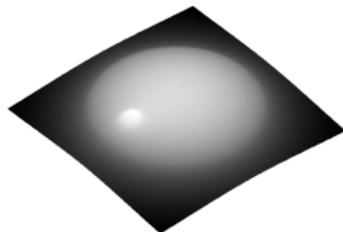
8 projections



16 projections



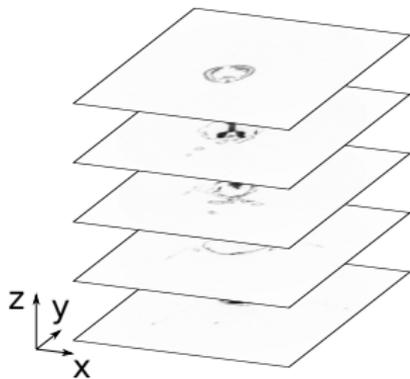
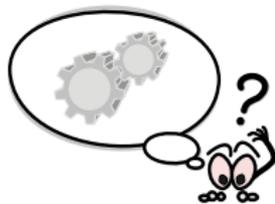
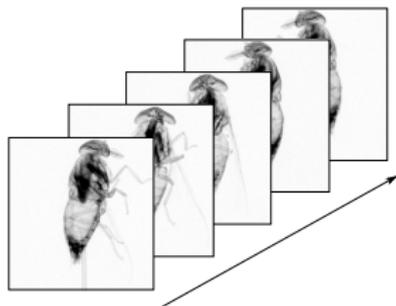
32 projections



The solution is too smooth. . . something is missing!!!

Reconstruction

The scanning provides projection data. . .

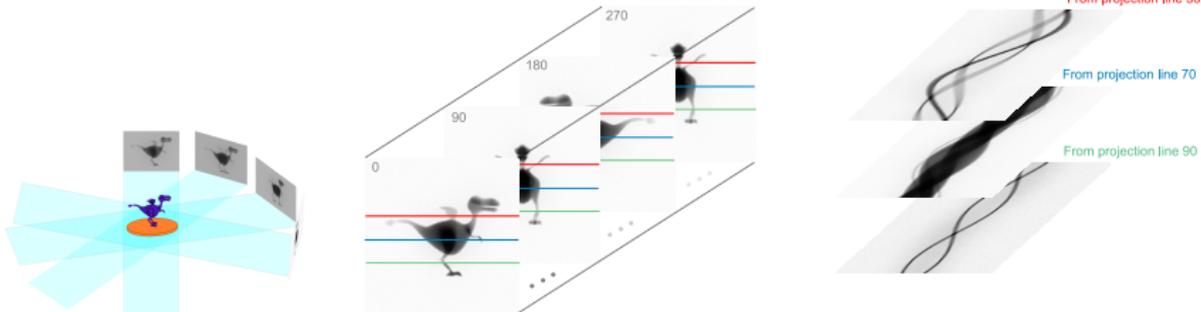


. . . but we want to find the cross section which caused the projection.

We have to find the inverse Radon transform or solve the equation system $Ax = y$

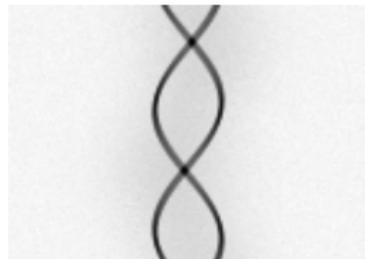
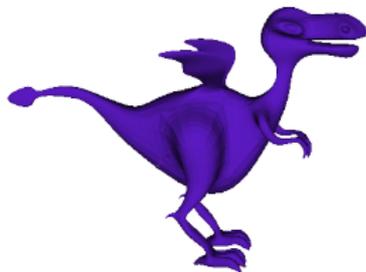
Sinogram construction

Combine take the same line from all projections into a new image

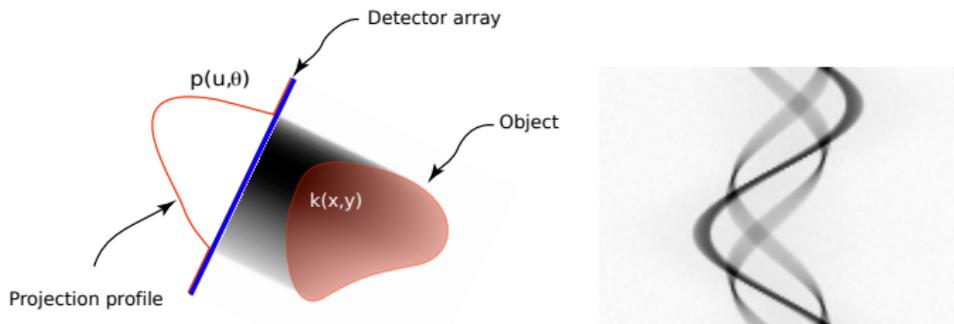


The information required to reconstruct a single slice.

Looking at the sinogram



Projection and sinogram



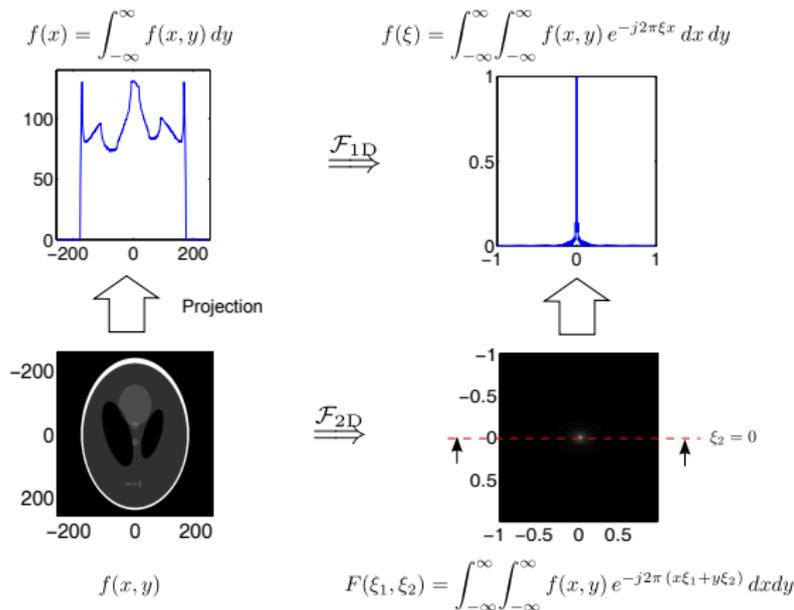
The Radon transform

An analytical description of projection I acquired at angle θ

$$p = \underbrace{-\ln\left(\frac{I(u, \theta)}{I_0(u)}\right)}_{\text{Measured}} = \int_{-\infty}^{\infty} \underbrace{k(x, y)}_{\text{Wanted}} \underbrace{\delta(x \cos \theta + y \sin \theta - u)}_{\text{Observation ray}} dx dy$$

Theorem

The Fourier transform of a parallel projection $p(x)$ of an object $f(x, y)$ obtained at an angle θ equals a line through origin in the 2D Fourier transform of $f(x, y)$ at the same angle.

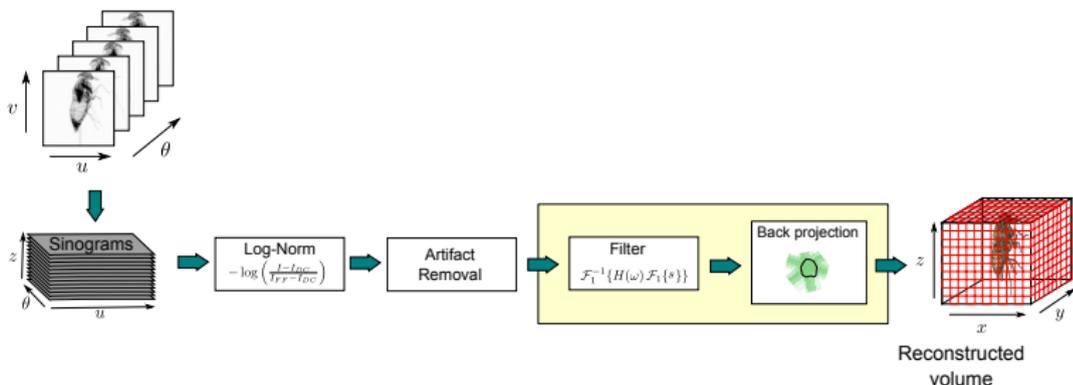


Reconstruction in the frequency domain

$$k(x, y) = \int_0^\pi \int_{-\infty}^{\infty} |\omega| P(\omega, \theta) e^{j2\pi\omega(x \cos \theta + y \sin \theta)} d\omega d\theta$$

Reconstruction in the spatial domain

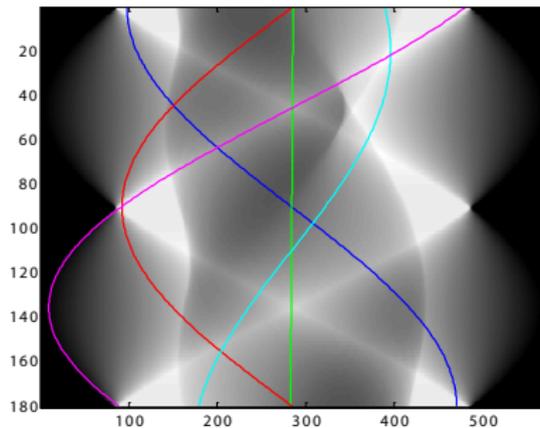
$$k(x, y) = \frac{1}{2\pi^2} \int_0^\pi \int_{-\infty}^{\infty} \underbrace{\partial p / \partial u(u, \theta)}_{\text{Convolution}} \underbrace{[x \cos \theta + y \sin \theta - u]^{-1}}_{\text{Rotation}} du d\theta$$



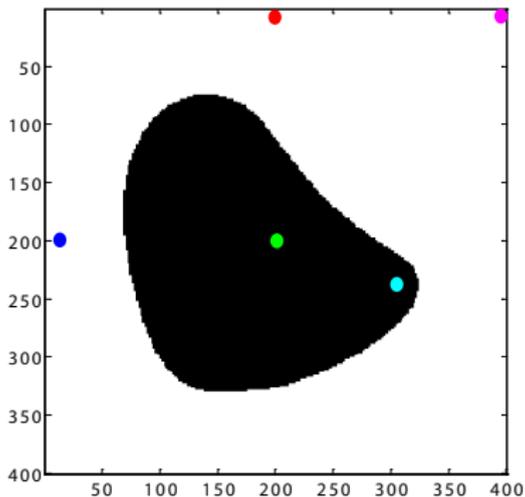
Some line integrals in the sinogram

The value of a single pixel is given by the line integral along a sine.

Sinogram



Cross section



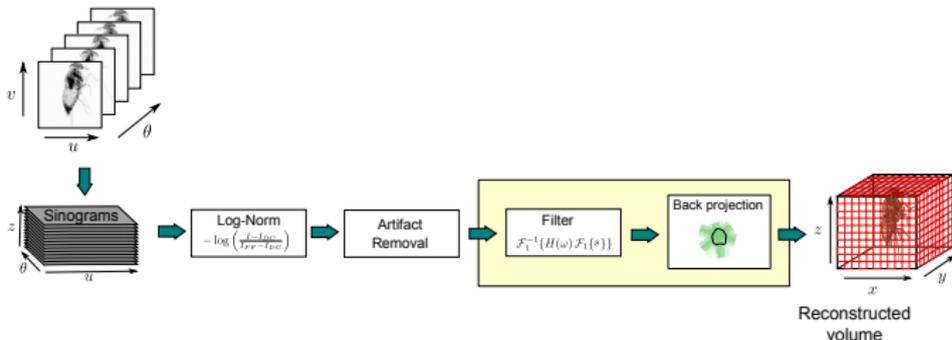
Reconstruction in the spatial domain

$$k(x, y) = \frac{1}{2\pi^2} \int_0^\pi \int_{-\infty}^{\infty} \underbrace{\frac{\partial p}{\partial u}(u, \theta)}_{\text{Convolution}} \underbrace{[x \cos \theta + y \sin \theta - u]^{-1}}_{\text{Rotation}} du d\theta$$

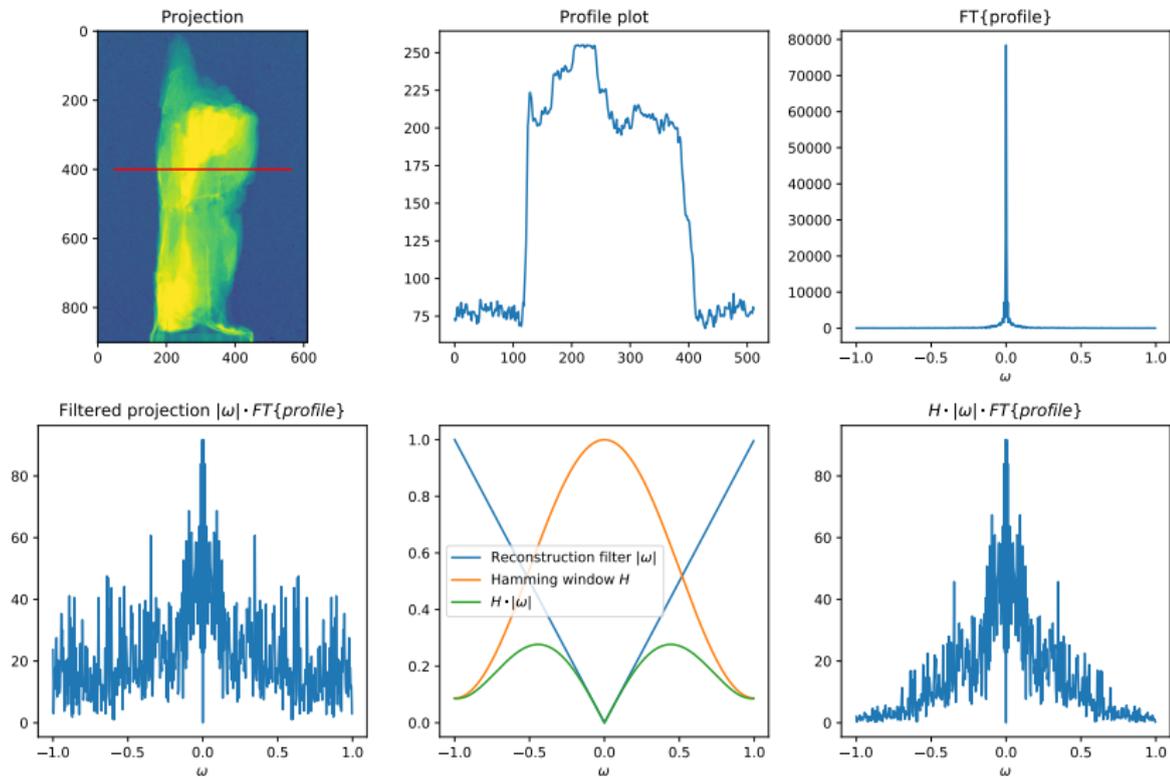
The filter

The filter has two components:

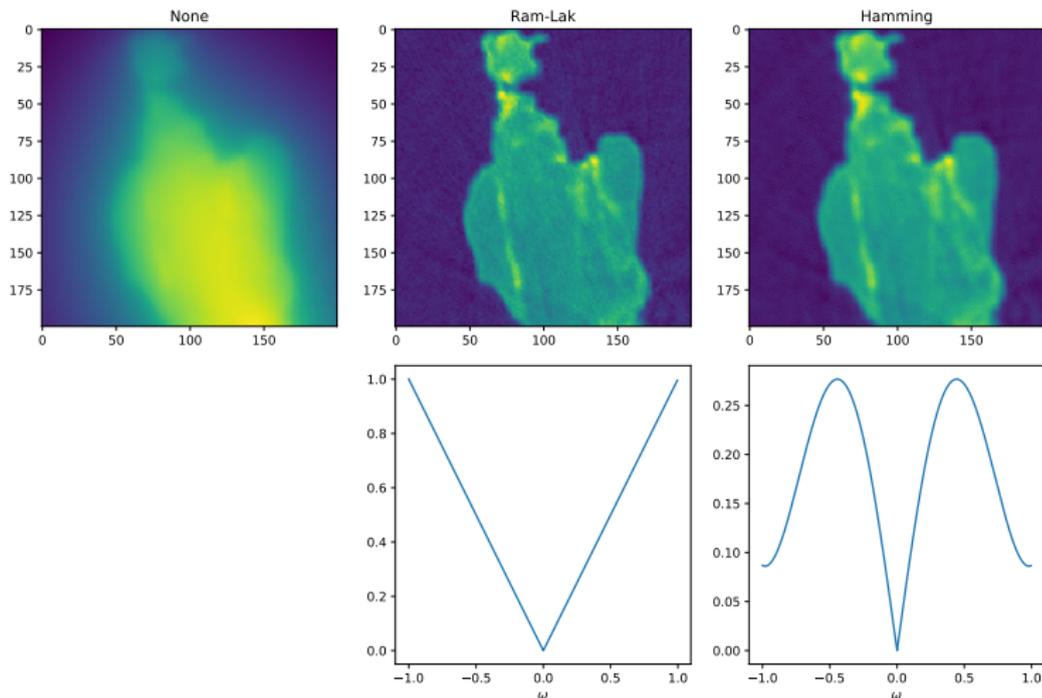
- A derivative: $\partial p / \partial u(u, \theta) \equiv \mathcal{F}^{-1}(|\omega| \cdot \mathcal{F}(p))$
- Apodization: Shepp-Logan, Hamming, *etc*



Reconstruction filter in action



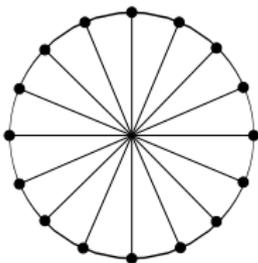
The effect of the reconstruction filter



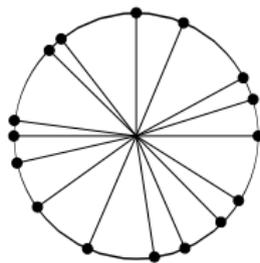
Apodization filters suppress noise and blur edges

When the analytical solution has problems

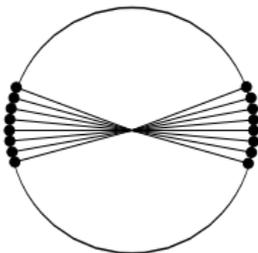
Few projections



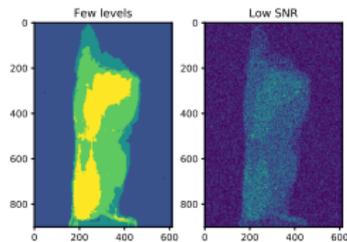
Irregularly distributed



Limited view



Low SNR or contrast



Algebraic methods

- ART
- SIRT
- TV
- etc...

Statistical methods

- Maximum likelihood
- Penalized ML
- etc..

Pros & cons

- + Sparse, irregularly sampled projection data
 - Limited angle
 - Few views
- + Physical model can be included
- Requires prior information for best performance.
- Time consuming

Building the system matrix

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + \dots &= y_1 \\
 a_{21}x_3 + a_{22}x_4 + \dots &= y_2 \\
 a_{31}x_1 + a_{32}x_3 + \dots &= y_3 \\
 &\vdots
 \end{aligned}
 \quad
 \begin{bmatrix}
 a_{11} & \cdots & a_{1N} \\
 \vdots & \ddots & \vdots \\
 a_{N1} & \cdots & a_{NN}
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 \vdots \\
 x_N
 \end{bmatrix}
 =
 \begin{bmatrix}
 y_1 \\
 \vdots \\
 y_N
 \end{bmatrix}$$

Example

You have:

- 1000 projections which are 1000 pixels wide
- The reconstructed slice has 1000×1000

This gives $1000 \times 1000 \times 1000 = 10^9$ equations

→ A is a $10^9 \times 10^9$ matrix!

- Sparse matrix
- Ill-posed (ideally infinitely many equations needed)
- Inversion doesn't provide unique solution

Problem to solve

We want to solve the equation $Ax = y$,
where A is the forward projection operator, a large sparse matrix

Kaczmarz method (ART)

$$x^{k+1} = x^k + \lambda_k \frac{y_i - \langle a_i, x^k \rangle}{\|a_i\|^2} a_i$$

a_i the i^{th} row of the system matrix A .

x^k the reconstructed image at the k^{th} iteration.

y_i the i^{th} element of the sinogram

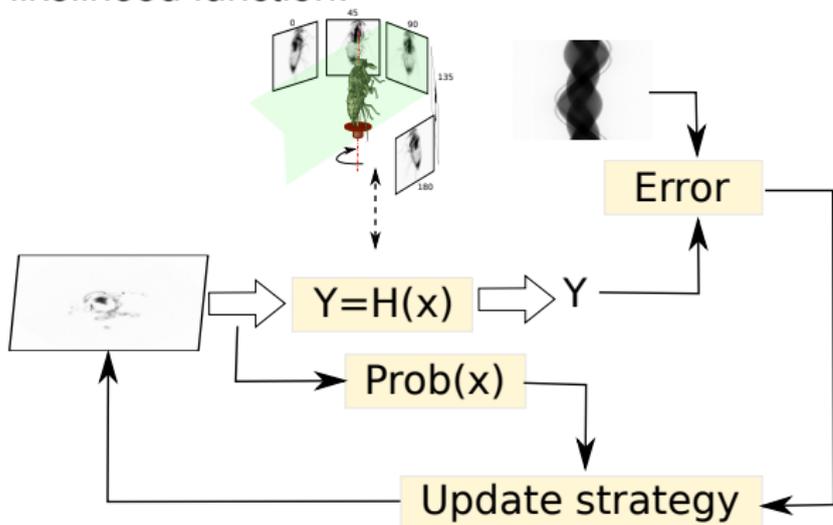
λ_k relaxation parameter

Problem to solve

We want to solve the equation $Ax = y + noise$,

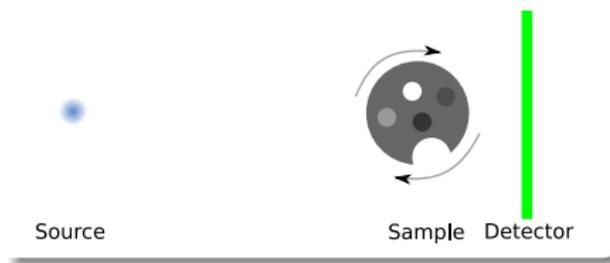
Iteration scheme

Maximize likelihood function:

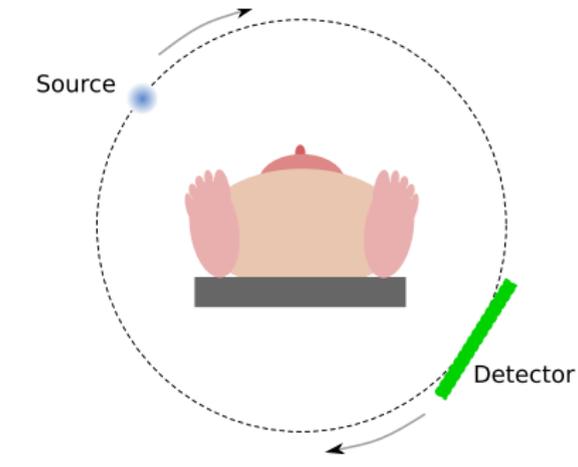


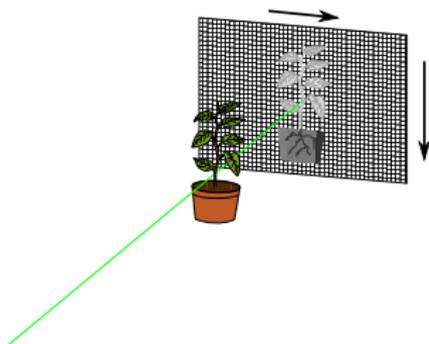
Beam geometry

Static beamline

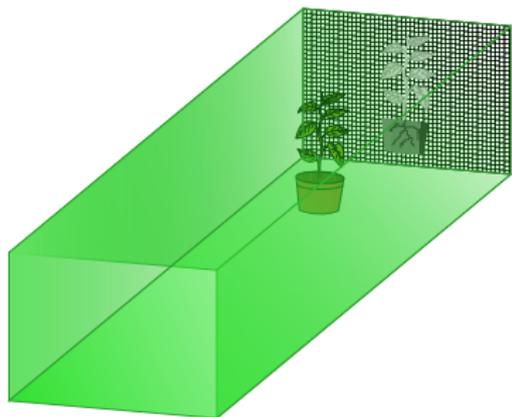


Rotating beamline

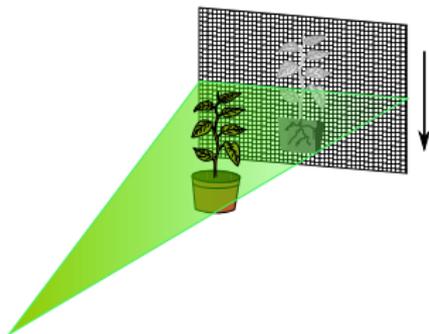




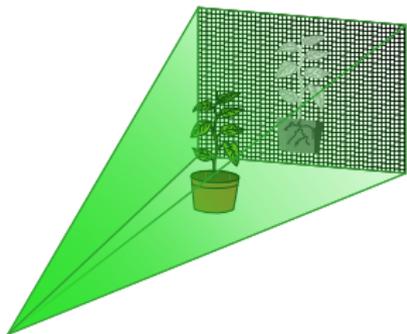
- Simple beam geometry
- Single pixels are scanned
- The 'Hounsfield-approach'



- Produces 2D projections
- No geometric unsharpness
- Simple reconstruction, filtered back projection [Buzug, 2008]

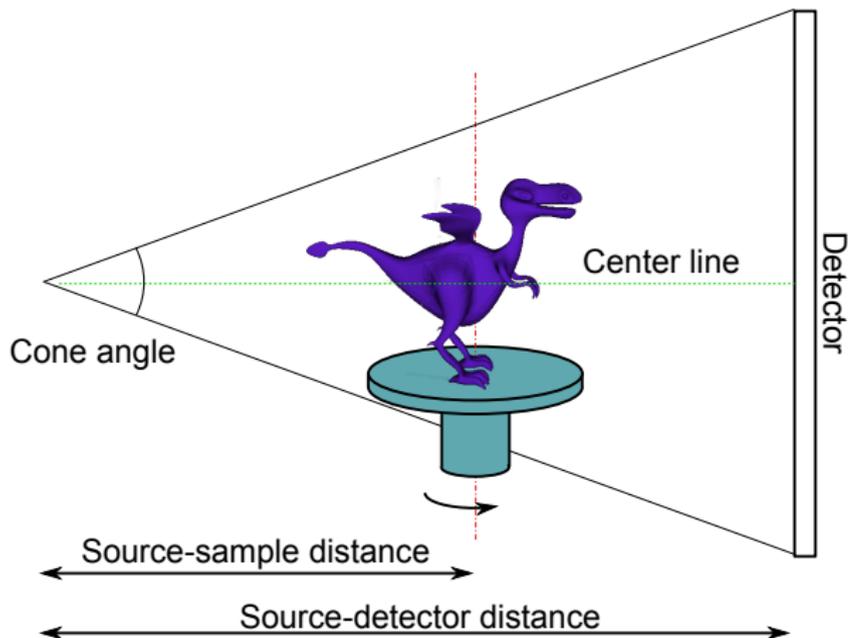


- Line-wise scan
- Beam incidence must be perpendicular to detector plane
- Magnifying in one direction

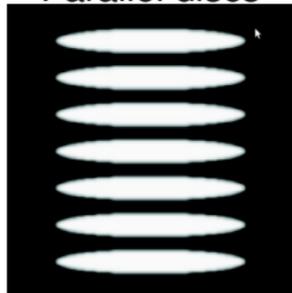
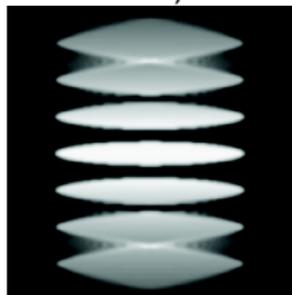


- + Uses 2D-projections.
- + Magnifying due to beam divergence.
- Non-trivial reconstruction using [Feldkamp et al., 1984].
- Only in the central slice is exact.

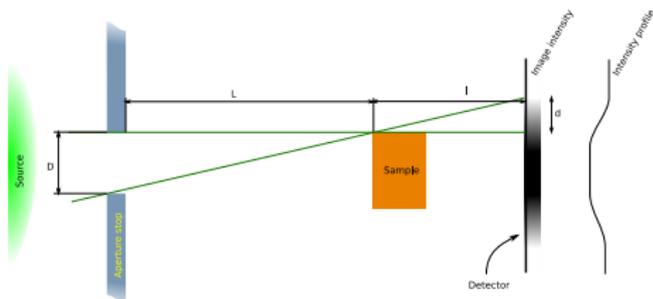
Problems with cone-beam



Parallel discs

FDK (cone angle
30°)

Penumbra blurring



Collimation ratio

The width of the penumbra blurring is described by the collimation ratio:

$$\frac{L}{D} = \frac{l}{d}$$

L Distance from aperture to sample

D Width of aperture diameter

l Distance from sample to detector

d Width of unsharpness

Typical collimation ratio $L/D = 100 - 2000$ [mm/mm]

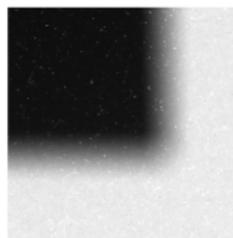
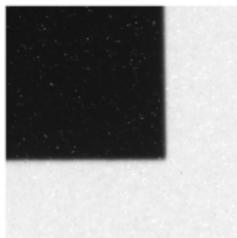
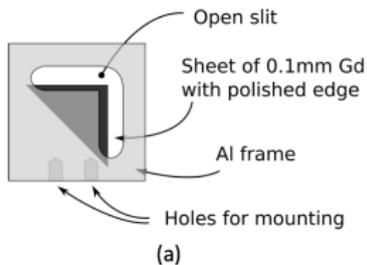


Fig 3. Schematic of the edge sample (a) and neutron radiographs obtained with the sample at 3mm (b) and 320mm (c) away from the detector. The edge unsharpness is mainly caused by penumbra blurring.

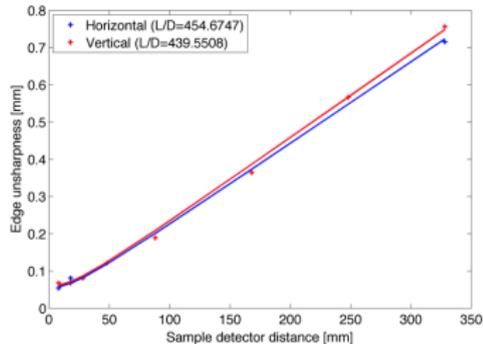
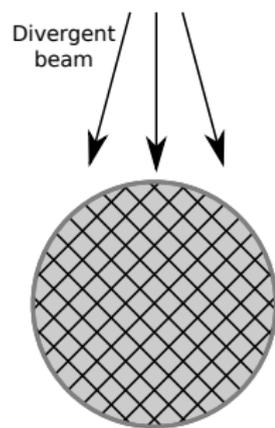
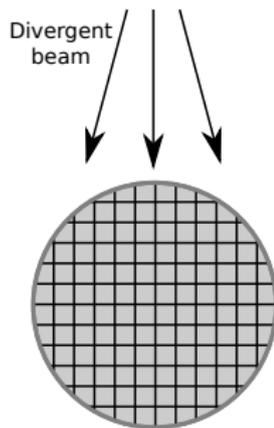
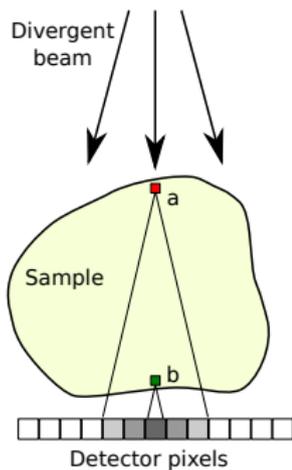
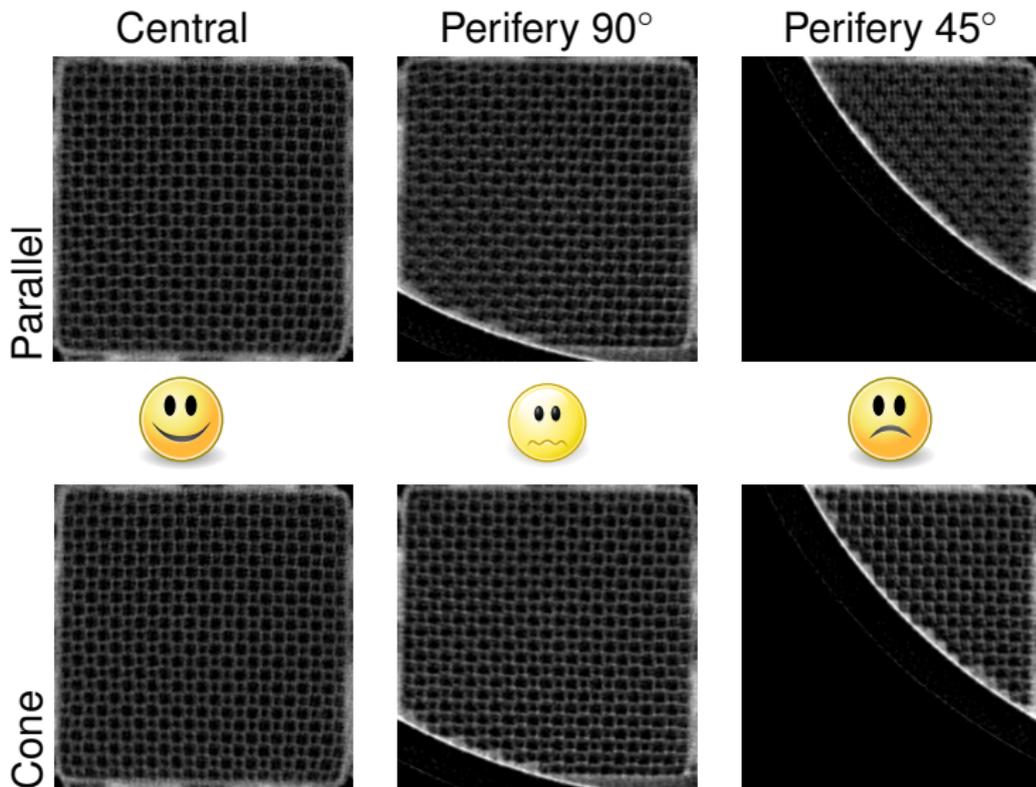


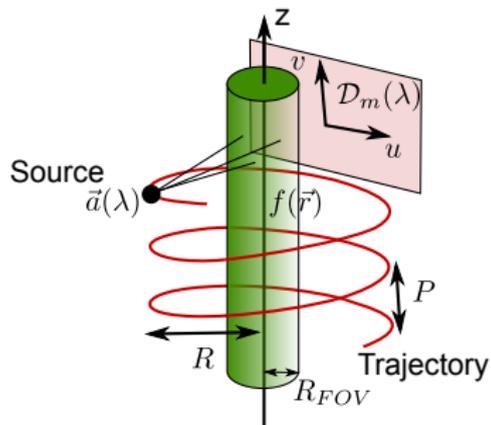
Fig 4. It is possible to estimate the collimation ratio by measuring the edge unsharpness at different distances from the detector.

The impact of beam divergence



Improved results using CBCT reconstruction

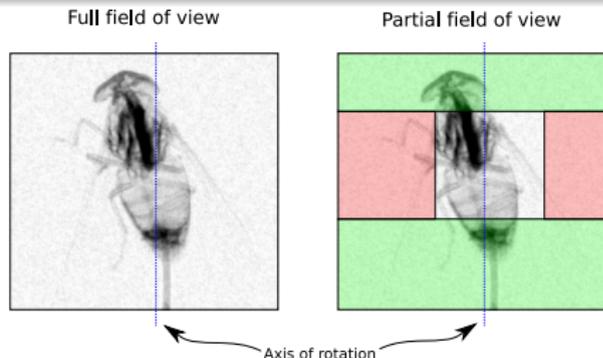




- Exact 3D solution
- Long objects
- Reconstruction using Katsevich[Katsevich, 2002]

Requirement

Projections from at least 180° + sample must always be visible.

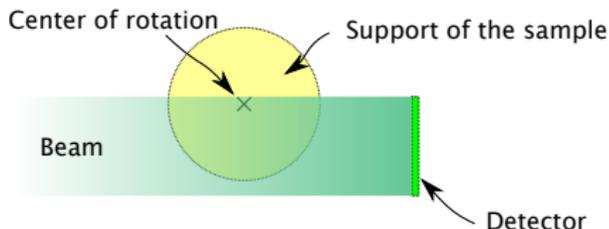


Two options to handle samples larger than the field of view

- Translate the COR and use a 360° orbit.
- Truncated reconstruction

Idea

- Translate the COR to the side of the projection
- Near doubled FOV



Requirements

- The projections must be stitched
- Projections must be acquired over 360°
- More voxels requires more projections

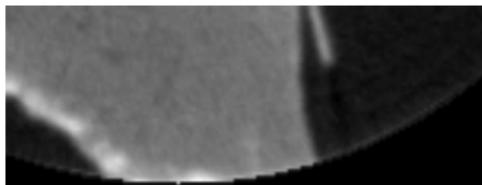
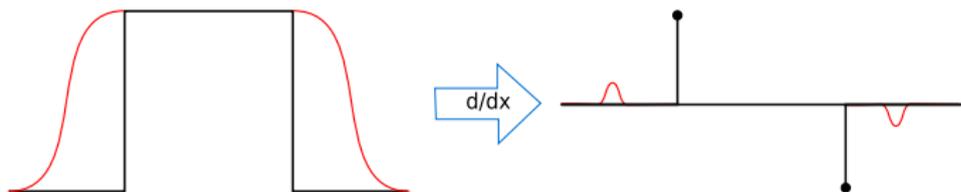
A truncated tomography has incomplete data support.

Effects of truncation

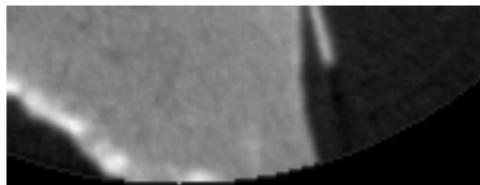
- 1 Some attenuation information is missing → bias
The shadow contains more attenuation than the projection data shows.
- 2 Truncation gives spikes on the edges.
The derivative in the reconstruction formula produce edge artifacts.

Origin The derivative of the truncated edge is steep

Solution Add a smooth transition from edge to zero



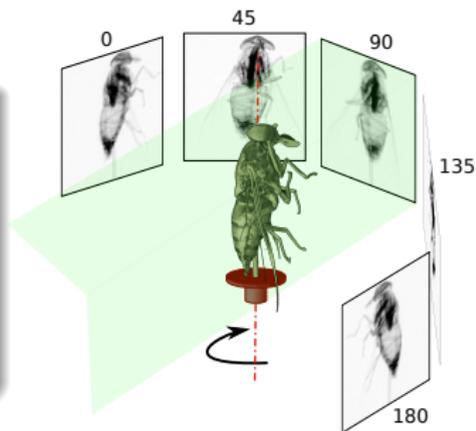
Original



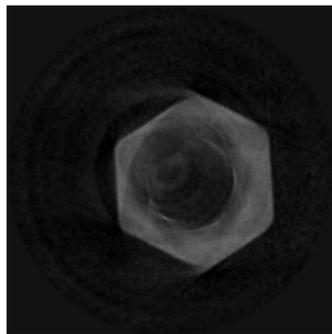
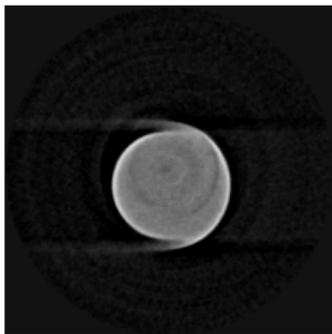
Padded

The axis

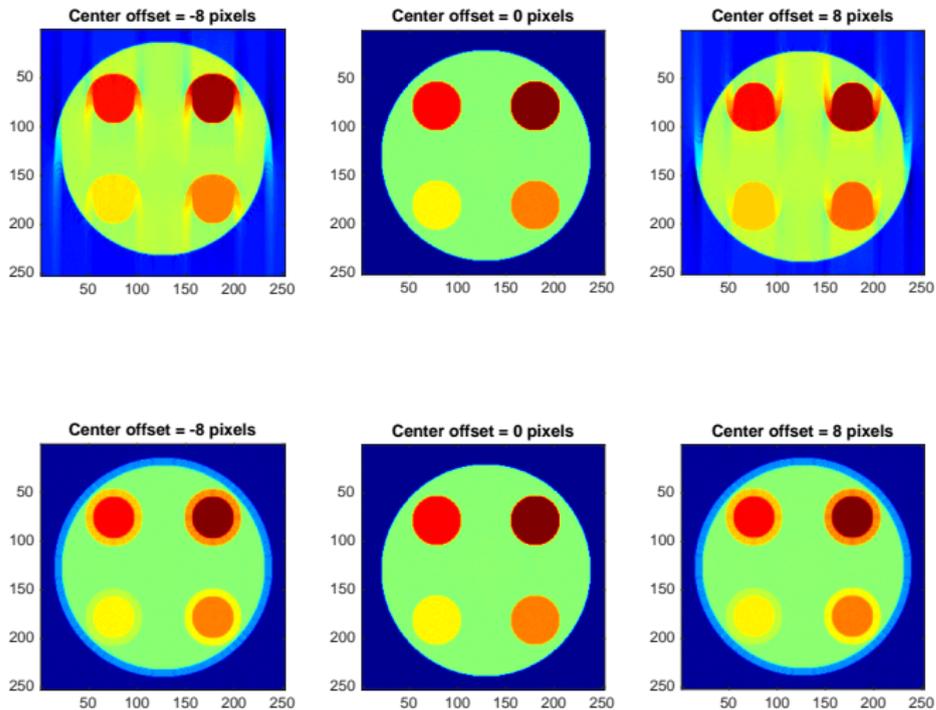
The point where all rays intersect is called the center of rotation for a single slice or the rotation axis for many slices. This point must be provided to the reconstructor.



Centering artifacts



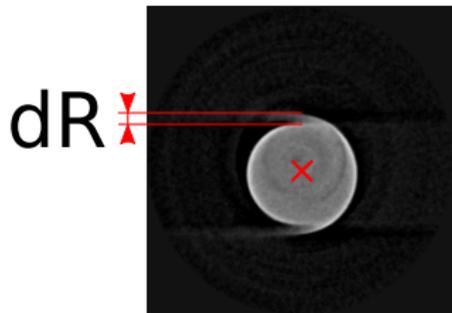
The impact of center misalignment



Projection data

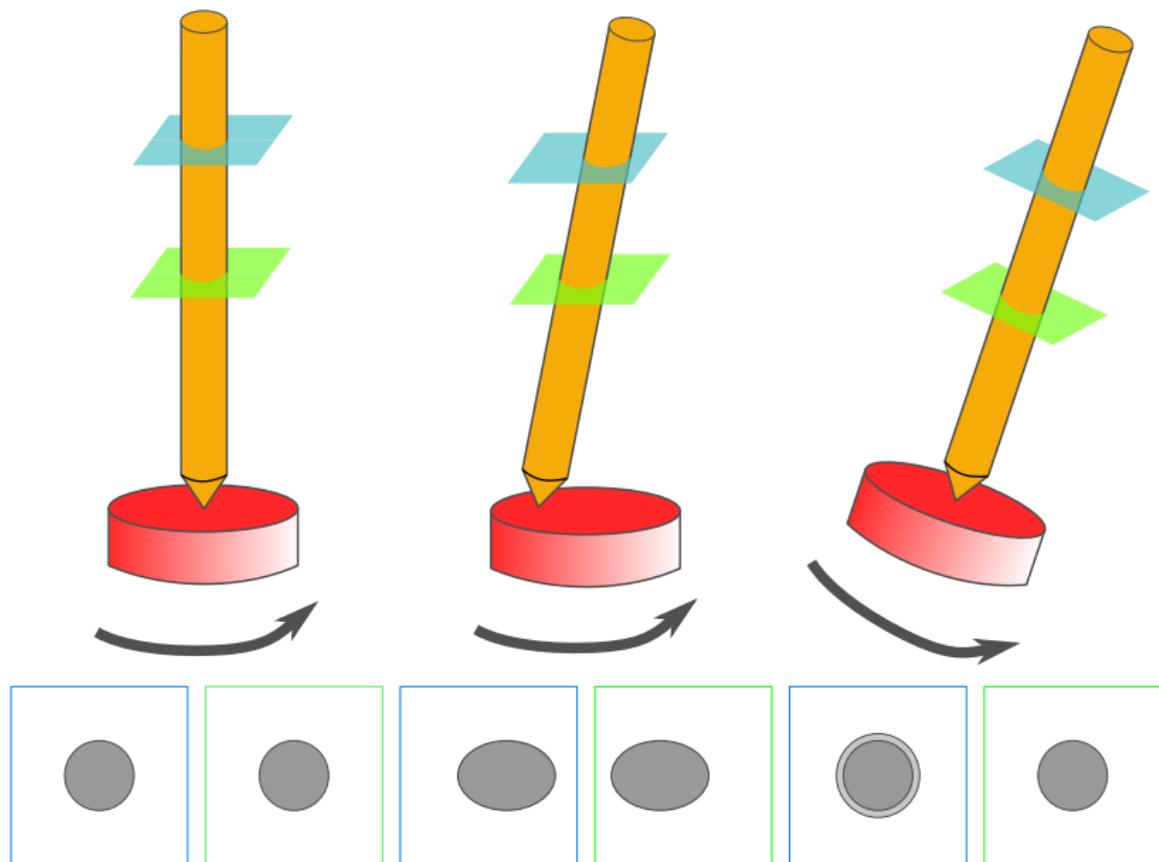
- Mirror one projection
- Translate until they overlap
- Center = midpoint + translation distance

Reconstructed data

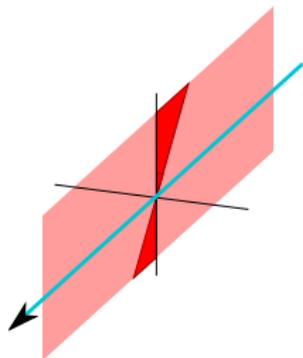


$$\text{center} = \text{current} \pm dR/2$$

Tilted sample or table

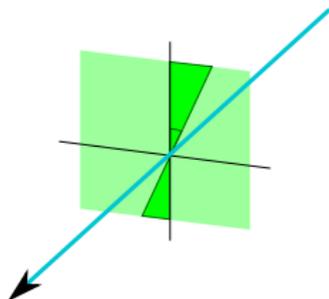


Along the beam



- Hard to correct
- Requires vector based reconstructor and geometry

Across the beam

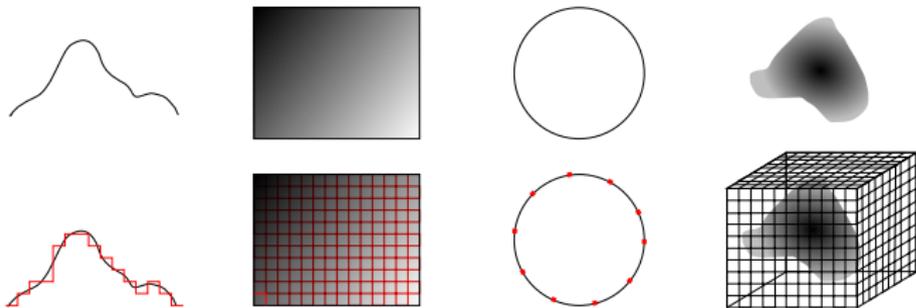


- Small angles corrected with COR shifts
- Large angles corrected with rotation

Sampling

The inversion formula is impractical since it would require infinite amount of equations to solve.

- The projections are digital images
 - Intensity sampling [bits/pixel]
 - Spatial sampling [pixels/mm]
- The rotation is done in steps
- The reconstruction is done on a finite matrix

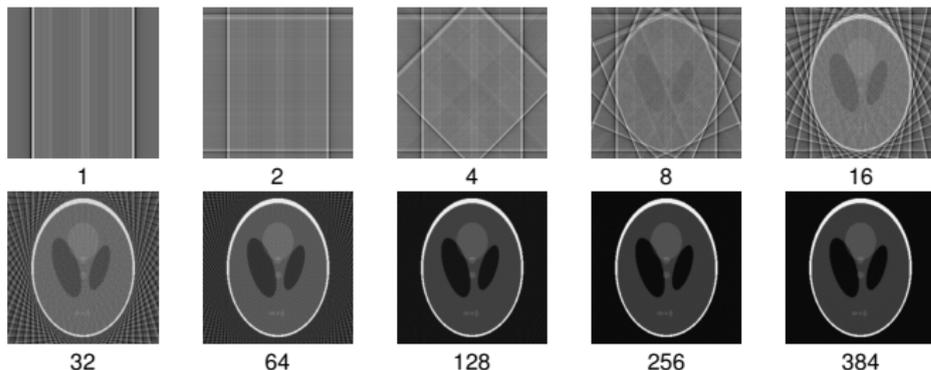


How many projections are needed?

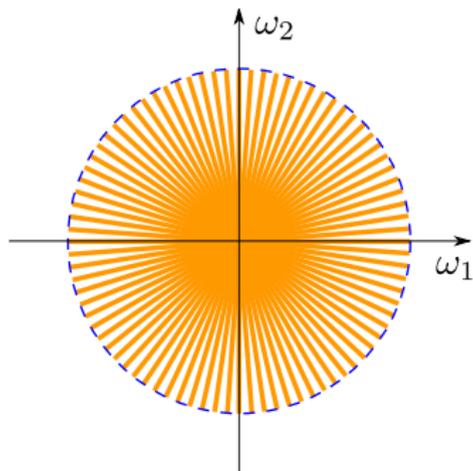
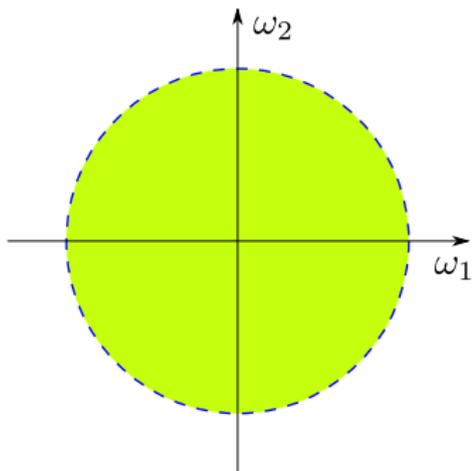
The number of projections is determined by the sampling theorem [Buzug, 2008].

$$N_{\text{projections}} = \frac{\pi}{2} N_U$$

N_U Number of pixels in the direction perpendicular to the axis of rotation.



Basic idea The unit circle in the Fourier domain must be filled.



Noise

Noise is an additive statistical phenomenon.

$$\mathcal{R}^{-1} \left\{ \begin{array}{c} \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \end{array} \right\} + \mathcal{R}^{-1} \left\{ \begin{array}{c} \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \end{array} \right\} = \mathcal{R}^{-1} \left\{ \begin{array}{c} \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \end{array} \right\} \rightarrow \begin{array}{c} \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \\ \text{[Image: Noisy CT slice]} \end{array}$$

Noise sources:

- Thermal noise from the electronics.
- Algorithmic, rounding errors, interpolation model etc.
- Noise induced by the radiation source.

Dose

The dose is the amount of radiation events hitting the detector.
More events improve the SNR (the law of great numbers).

The noise level of a slice is directly connected to the dose used.
The dose is defined as

$$Dose = Flux \times Time$$

The signal to noise ratio can be improved by increasing

- the beam intensity,
- the exposure time,
- the number of projections,
- detector exchange.

What influences the contrast?

$$C_{slice} W_{sample} \sim C_{projection} N_{projections}$$

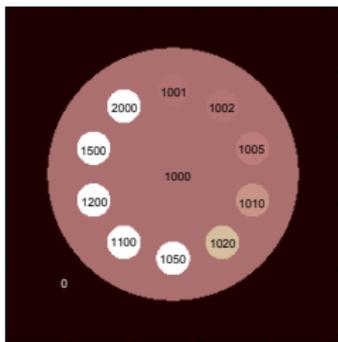
C_{slice} Slice contrast

$C_{projection}$ Projection contrast (Open beam - darkest region)

$N_{projections}$ Number of projections

W_{sample} Largest width of the sample in pixels

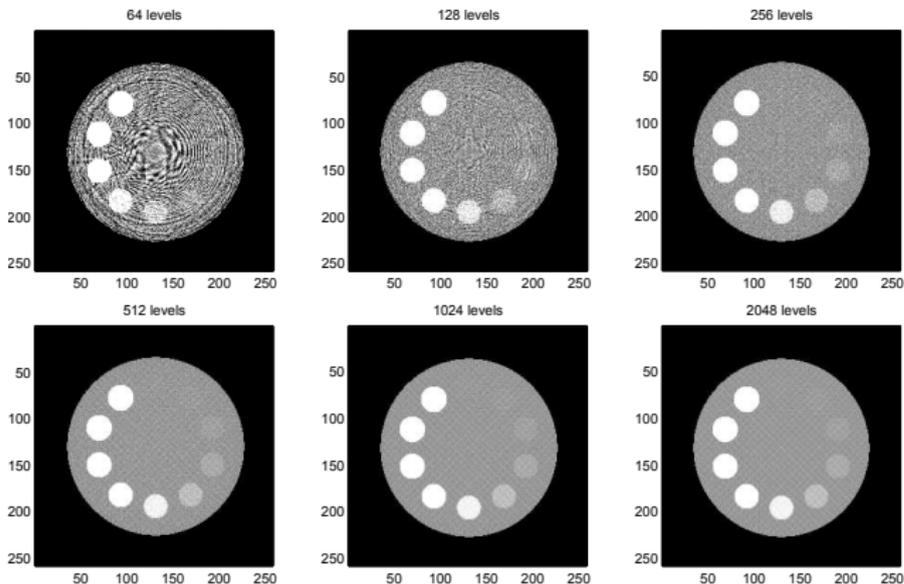
The phantom



Parameters

- $w=192$
- $N_{projections}=288$
- $C_{projection}=6, 7, 8, 9, 10, 11, 12, 13$ bits
- Contrast ratio: 1000:1, ..., 1:2
- Noise free

Changing projection contrast with constant number of projections



The reconstruction noise decrease with increasing dynamics

Artefacts

Rings are caused by stuck or dead pixels. They have the same value for all projections

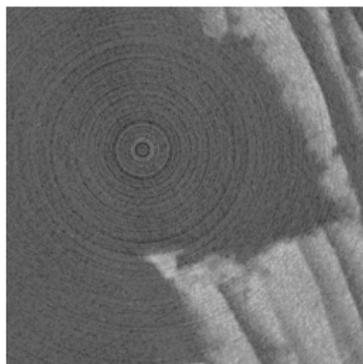
Lines are caused by single pixels or groups pixels in a single projection

High contrast these artifacts appear as star-like streaks originating from the high contrast object.

Motion when the sample changes during acquisition.

Beam hardening Polychromatic beam

Scattering The beam is scattered



- Ring artefacts are very common in tomography.
- They are caused by a stuck, dead, or hot pixels.
- They appear as:
 - Lines in the sinogram
 - Concentric rings in the CT slices

Projections Identify and remove spots that persists through projections.

Sinograms Identify lines parallel to the θ -axis

- Subtract first derivative of average projection from sinogram.
- Filter sinogram in Fourier domain (notch filter or wavelet filter).

$$s(u, \theta) - \mathbf{1}^T \cdot (E_u[s] - \text{median}_N(E_u[s])) =$$

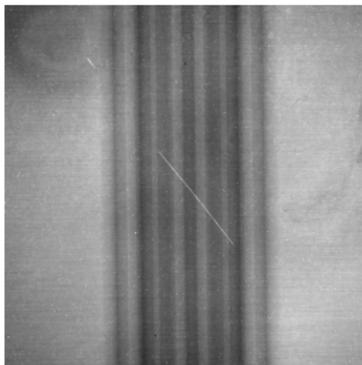
Correction procedure:

- Transform matrix to polar coordinates
- Detect lines
- Make replacement map
- Transform map to Cartesian coordinates
- Correct matrix

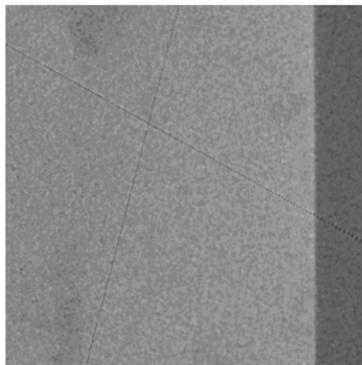
Advantage Good for testing different strengths

Disadvantage The coordinate transformations

Line artifacts



Projection



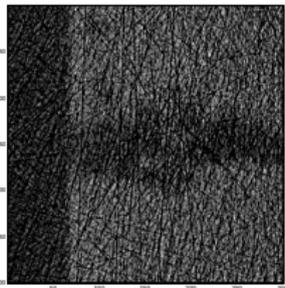
Reconstructed slice

- Line artifacts are more common with neutrons
- The origin of a line is a local spot in the sinogram.
- The orientation and position depends on when the spot was registered.

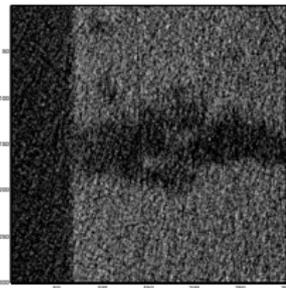
Correction method

- Detect the spots on the projections – compute local variances
- Replacement *e.g.*

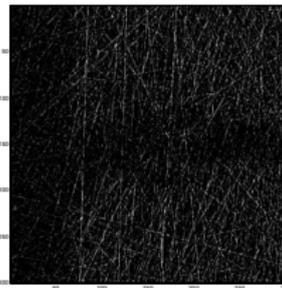
$$p_{corrected} = w(\sigma) \cdot p + (1 - w(\sigma)) \cdot p_{median} \text{ with } 0 \leq w \leq 1$$



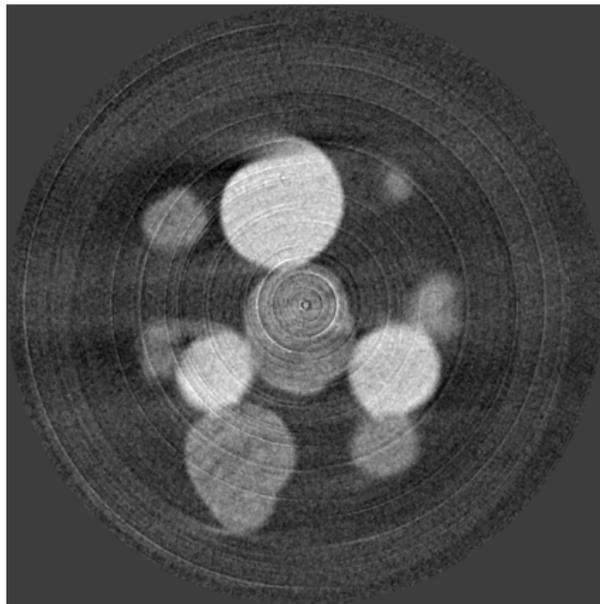
Raw



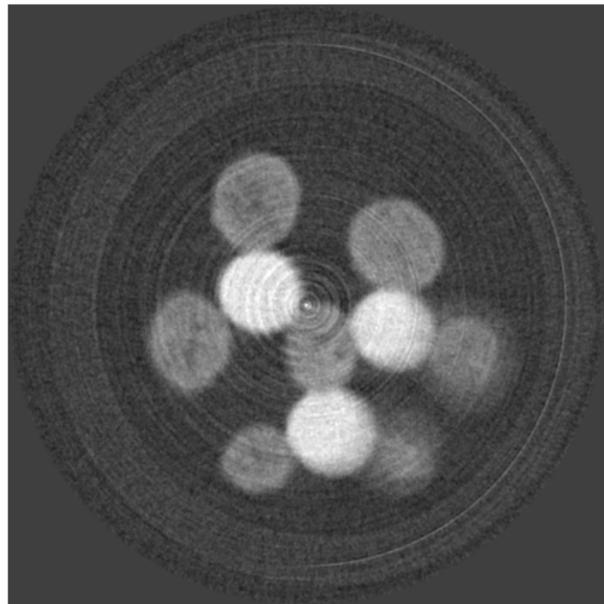
Corrected



Difference



Sequential acquisition



Golden ratio acquisition

Dynamic processes are hard to observe with CT

- CT needs long scan times.
- If the interfaces move more than 1 pixel during the scan motion artifacts will appear.

The solution

- Increment the acquisition angle by the Golden ratio $\phi = \frac{1+\sqrt{5}}{2}$
- The sample will always be observed at 'orthogonal' angles.

Definition

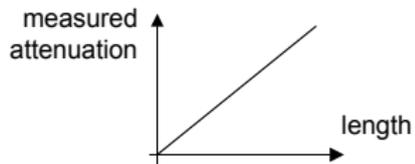
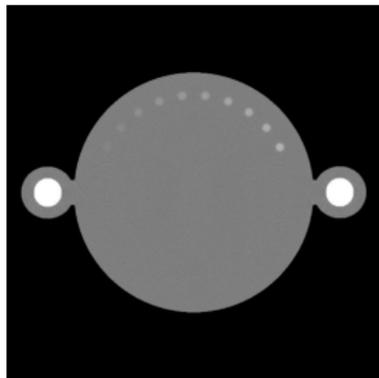
Cupping is a phenomenon that appears as a drop of attenuation coefficients in large homogeneous bodies. The main origins are:
Beam hardening when the radiation attenuation depends on energy.

Scattering background scattering adds a bias.

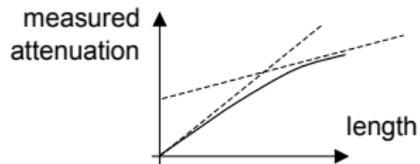
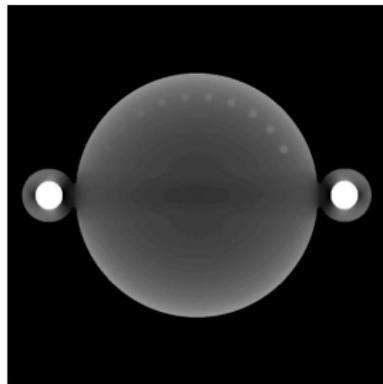
Cupping due to Beam hardening

The attenuation depends on energy

Monochromatic



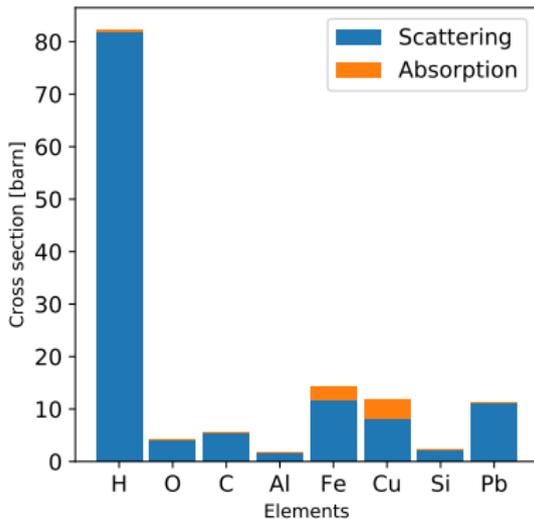
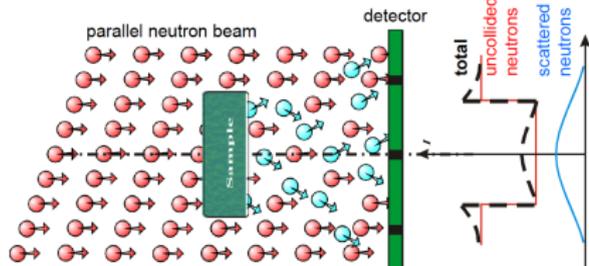
Polychromatic



The attenuation law assumes the intensity to be absorbed. . .

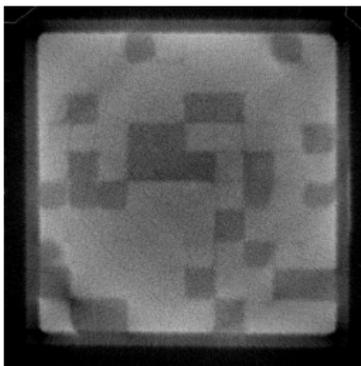
This is not true for neutrons!!!

Most neutrons are scattered

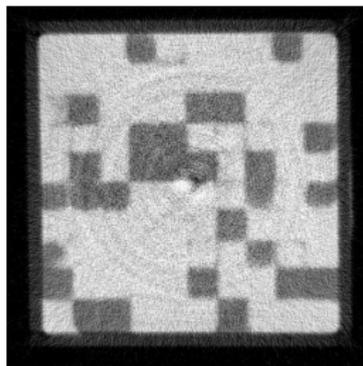


Scattered neutrons are bad for

- Quantitative imaging
- Segmentation algorithms

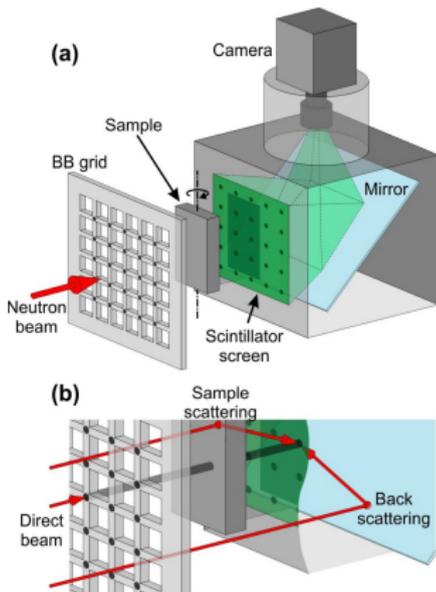


Uncorrected



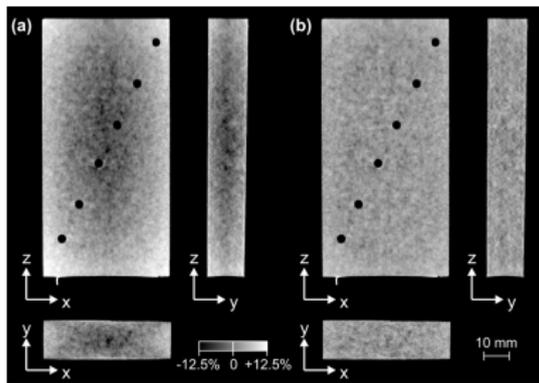
Corrected by QNI [Hassanein, 2006]

Measurements



Correction and result

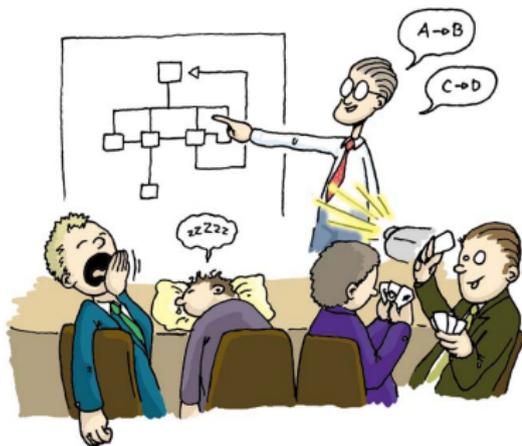
- Estimate scattering profile using black bodies.
- Correction using revised projection normalization



Summary

- Tomography is an indirect acquisition method
- Different sources can be used
- The perfect tomography needs
 - many projections
 - well illuminated projections
- Artifacts may and will appear but can mostly be corrected.

I'm done



Your turn

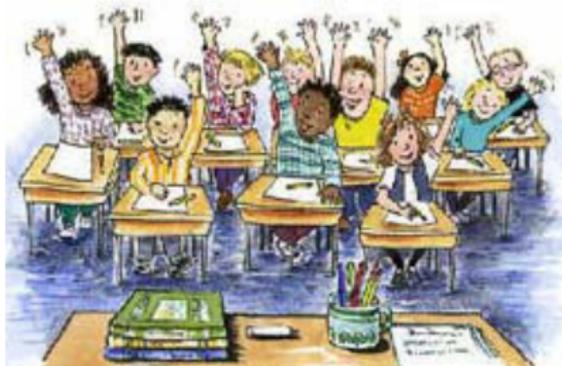


Image function:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\xi_1, \xi_2) e^{j2\pi(x\xi_1 + y\xi_2)} d\xi_1 d\xi_2$$

Coordinate transform $\{\xi_1, \xi_2\} = \{\omega \cos \theta, \omega \sin \theta\}$

$$f(x, y) = \int_0^{2\pi} \int_{-\infty}^{\infty} F(\omega \cos \theta, \omega \sin \theta) e^{j2\pi\omega(x \cos \theta + y \sin \theta)} d\omega d\theta$$

Fourier slice theorem:

$$f(x, y) = \int_0^{2\pi} \int_{-\infty}^{\infty} P(\omega, \theta) e^{j2\pi\omega(x \cos \theta + y \sin \theta)} \omega d\omega d\theta$$

Symmetry properties:

$$P(\omega, \theta + \pi) = P(-\omega, \theta)$$

Rotated coordinates:

$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{\infty} P(\omega, \theta) e^{j2\pi\omega(x \cos \theta + y \sin \theta)} |\omega| d\omega d\theta$$

A basic back-projection algorithm

pProj : pointer to line in sinogram
 pSlice : pointer to slice matrix

```

for (float line=0; line<nProjections; line++) {           // Loop over projections in s
  for (size_t y=0; y<SizeY; y++) {                       // Loop over matrix in y
    const size_t cfStartX = mask[y].first;              // Get x-coordinates
    const size_t cfStopX  = mask[y].second;
    fStartU += cos(theta[line]);                        // Compute first proj. pos.
    float fPosU=fStartU-sin(theta[line])*cfStartX;

    for (size_t x=cfStartX; x<cfStopX; x++) {           // Loop over matrix in x
      int nPosU=static_cast<int>(fPosU-sin(theta[line])); // Compute position

      const float interpB=fPosU-nPosU;                  // Interpolation weight right
      const float interpA=1.0f-interpB;                 // Interpolation weight left

      pSlice[x+y*sizeX]+=interpA*pProj[nPosU]+interpB*pProj[nPosU+1];
    }
  }
}

```



Boillat, P., Carminati, C., Schmid, F., Grünzweig, C., Hovind, J., Kaestner, A., Mannes, D., Morgano, M., Siegwart, M., Trtik, P., Vontobel, P., and Lehmann, E. (2018).
Chasing quantitative biases in neutron imaging with scintillator-camera detectors: a practical method with black body grids.
Optics Express, 26(12):15769.



Bracewell, R. (1956).
Strip integration in radio astronomy.
Australian Journal of Physics, 9(2):198.



Buzug, T. (2008).
Introduction to Computed Tomography: From photon statistics to modern cone-beam CT.
Springer.



Feldkamp, L., Davis, L., and Kress, J. (1984).
Practical cone-beam algorithm.
J. Opt Soc. Am. A, 1(6):612–619.



Hassanein, R. (2006).
Correction methods for the quantitative evaluation of thermal neutron tomography.
Diss. eth no. 16809, Swiss Federal Institute of Technology.



Kaestner, A., Grünzweig, C., and Lehmann, E. (2012).
Improved analysis techniques for new applications in neutron imaging.
In *Proc 18th World Conference on Nondestructive Testing*, pages 1–10.



Kaestner, A., Kis, Z., Radebe, M., Mannes, D., Hovind, J., Grünzweig, C., Kardjilov, N., and Lehmann, E. (2017).
Samples to determine the resolution of neutron radiography and tomography.
Physics Procedia, 88:258–265.



Kaestner, A., Münch, B., Trtik, P., and Butler, L. (2011).
Spatio-temporal computed tomography of dynamic processes.
Optical Engineering, 50(12):123201.



Katsevich, A. (2002).
Theoretically exact filtered backprojection-type inversion algorithm for spiral ct.
SIAM Journal on Applied Mathematics, 62(6):2012–2026.



Köhler, T. (2004).
A projection access scheme for iterative reconstruction based on the golden section.
In *Nuclear Science Symposium Conference Record*, volume 6, pages 3961–3965. IEEE.



Radon, J. (1917).
Ueber die bestimmung von funktionen durch ihre Integralwerte tangs gewisser mannigfaltigkeiten.
Berichte Saechsisch Akad. Wissenschaften (Leipzig), Math. Phys., Klass 69:262–277.



Sears, V. (1992).
Neutron scattering lengths and cross sections.
Neutron News, 3(3):26–37.